INTERIOR SCHEAUDER ESTIMATES FOR THE FOURTH ORDER HAMILTONIAN STATIONARY EQUATION IN TWO DIMENSIONS

ABSTRACT. (This is on a joint work with Micah Warren.)

We study the regularity of the Lagrangian Hamiltonian stationary equation, which is a fourth order nonlinear PDE. Consider the function $u : B_1 \to \mathbb{R}$ where $B_1$ is the unit ball in $\mathbb{R}^2$. The gradient graph of $u$, given by $\{(x,Du(x))|x \in B_1\}$ is a Lagrangian submanifold of the complex Euclidean space. The function $\theta$ is called the Lagrangian phase for the gradient graph and is defined by

$$\theta = F(D^2 u) = Im \log \det(I + D^2 u).$$

The non homogeneous special Lagrangian equation is given by the following second order nonlinear equation

$$F(D^2 u) = f(x).$$

The Hamiltonian stationary equation is given by the following fourth order nonlinear PDE

$$\Delta_g \theta = 0$$

where $g$ is the induced Riemannian metric from the Euclidean metric on $\mathbb{R}^4$, which can be written as

$$g = I + (D^2 u)^2.$$

We consider the Hamiltonian stationary equation for all phases in dimension two and show that solutions that are $C^{1,1}$ will be smooth and we also derive a $C^{2,\alpha}$ estimate for it.