INTERIOR SCHAUDER ESTIMATES FOR THE FOURTH ORDER HAMILTONIAN STATIONARY EQUATION IN TWO DIMENSIONS

Abstract. (This is on a joint work with Micah Warren.)

We study the regularity of the Lagrangian Hamiltonian stationary equation, which is a fourth order nonlinear PDE. Consider the function $u : B_1 \to \mathbb{R}$ where B_1 is the unit ball in $\mathbb{R}^2$. The gradient graph of u, given by $\{(x, Du(x)) | x \in B_1\}$ is a Lagrangian submanifold of the complex Euclidean space. The function θ is called the Lagrangian phase for the gradient graph and is defined by

$$\theta = F(D^2u) = Im \log \det(I + iD^2u).$$

The non homogeneous special Lagrangian equation is given by the following second order non-linear equation

$$F(D^2u) = f(x).$$

The Hamiltonian stationary equation is given by the following fourth order nonlinear PDE

$$\Delta_g \theta = 0$$

where g is the induced Riemannian metric from the Euclidean metric on $\mathbb{R}^4$, which can be written as

$$g = I + (D^2u)^2.$$

We consider the Hamiltonian stationary equation for all phases in dimension two and show that solutions that are $C^{1,1}$ will be smooth and we also derive a $C^{2,\alpha}$ estimate for it.