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Singularities and dynamics of Mean Curvature Flow
Open Problems

1. The existence problem for self-shrinkers: There are a number of numerical examples of embedded self-shrinkers in $\mathbb{R}^3$, including examples of Chopp and Angenent–Ilmanen–Chopp. Prove rigorously that these examples exist. It may be easier to start by proving existence of immersed solutions (like Sacks–Uhlenbeck for minimal surfaces or Wente for cmc tori). Gluing may be fruitful.

2. The genus zero conjecture of Ilmanen: Prove that the round $\mathbb{S}^2$ is the only embedded self-shrinker in $\mathbb{R}^3$ that is a topological $\mathbb{S}^2$. Compare: Hopf problem for cmc surfaces.

   More generally, show that the flat plane and round cylinder are the only other genus zero examples.

   It is reasonable to assume some asymptotic structure in the latter cases.

3. Low values of the entropy: For $t_0 > 0$ and $x_0 \in \mathbb{R}^{n+1}$, define the Gaussian area functional F_{x_0,t_0} centered at x_0 and with scale t_0 by

   $$F_{x_0,t_0}(\Sigma) = (4\pi t_0)^{-\frac{n}{2}} \int_\Sigma \mathrm{e}^{-\frac{|x-x_0|^2}{4t_0}}.$$

   Define an invariant – which we call the *entropy* – λ by

   $$\lambda(\Sigma) = \sup_{x_0,t_0} F_{x_0,t_0}(\Sigma).$$

   Two conjectures:

   Conjecture: True in all dimensions: $\lambda(\Sigma^n) \geq \lambda(\mathbb{S}^n)$ for any closed hypersurface Σ.

   Conjecture: In $\mathbb{R}^3$, $\lambda(\mathbb{S}^1 \times \mathbb{R})$ is the third smallest (after $\mathbb{R}^2$ and $\mathbb{S}^2$).