Some Sharp Lower Density Bounds for Singularities of Minimal Varieties

What is the least possible density that a singularity in a minimal variety can have? I will describe some partial results (by Tom Ilmanen and myself) that are in a certain sense sharp.