Understanding the space of metrics with positive scalar curvature

The problem of whether or not a smooth manifold admits a metric of positive scalar curvature has been extensively studied. Indeed, in the case of closed simply connected manifolds of dimension at least five, this question is completely answered. Far less is known about the topology of the space of positive scalar curvature metrics, or of the corresponding moduli space (obtained by quotienting out by the action of the diffeomorphism group on $X$). In this talk I will discuss some recent progress in this subject, some of which is joint work with Boris Botvinnik, Bernhard Hanke and Thomas Schick.