A general regularity theory for stable codimension 1 integral varifolds

The focus of this talk will be a new regularity theorem for the class of singular stable minimal hypersurfaces (stable codimension 1 integral varifolds) in an open ball. Making no assumption a priori on the size of the singular set, the theorem gives a natural, geometric structural condition for a hypersurface in this class to be smooth and embedded in the interior up to a lower dimensional, generally unavoidable, singular set. Precisely, suppose that a hypersurface in this class has the property that no singular point has a neighborhood in which the hypersurface is a union of $C^{1,\alpha}$ (for some arbitrarily chosen $\alpha \in (0, 1)$) hypersurfaces-with-boundary meeting (only) along their common boundary. Then it is smooth and embedded away from the boundary of the ball and away from a possible interior singular set of codimension at least 7 (which is empty if the dimension of the hypersurface is $\leq 6$). The work generalizes the regularity theory of R. Schoen and L. Simon. Some applications as well as what can be said in the absence of the above structural condition will also be discussed.