SOME RIGIDITY RESULTS ON THE HEMISPHERE

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The open problem related to my talk is to prove or disprove the following

Conjecture 0.1 (MinOo). *Let (M^n, g) be a smooth compact Riemannian manifold with boundary and scalar curvature $R \geq n(n-1)$. If the boundary is isometric to S^{n-1} and totally geodesic, then (M^n, g) is isometric to the hemisphere S^n_+.*

Partial results have been derived in [1, 2, 3]. More generally it would be interesting to formulate a notion for the positive curvature case corresponding to the concept of asymptotically flat manifolds for the case of comparison with the Euclidean spaces and prove an analogy of the positive mass theorem ([4]), which would imply Conjecture 0.1. Such a formulation should also enable us to get local comparison results in the positive curvature cases similar to those derived in [5].

References

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