OPEN PROBLEMS:

ON A GLUING CONSTRUCTION REGARDING "SMALL BODIES" IN GENERAL RELATIVITY

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1. There are several unexplored questions about the gluing presented in this talk. For example, it is not yet been investigated if one could do the following:

(1) Gluing analogous to the one presented in the talk, but with compact initial data replaced by an asymptotically euclidean or an asymptotically hyperbolic initial data.

(2) Localized gluing like that of J. Corvino [1] or Chruściel-Delay [2];

(3) Gluing of non-CMC initial data; good references for this topic are [3], [4] which rely on [5];

(4) Gluing of non-vacuum initial data; a good reference for this is [6].

2. In [7] S. Gralla and R. Wald describe a "small body" in general relativity via a smooth one-parameter family of space-time metrics $g(\lambda) = g(\lambda; x^\alpha)$ which obeys the following limit conditions:

(1) *Ordinary limit condition*: $g(0; x^\alpha)$ is smooth even at $r = \left|(x^1, x^2, x^3)\right| = 0$ while the curve $r = 0$ is timelike;

(2) *Scaled limit condition*: for each t_0 and the coordinate change $\bar{t} = \frac{t-t_0}{\lambda}$, $\bar{x}^\alpha = \frac{x^\alpha}{\lambda}$ (here $\alpha = 1, 2, 3$) each component $g_{\mu\nu}(\lambda; t_0, \bar{x}^\alpha)$ of g with respect to $(\bar{t}, \bar{x}^\alpha)$-coordinates is such that
$$\lambda^{-2} g_{\mu\nu}(\lambda; t_0, \bar{x}^\alpha)$$
is jointly smooth in $(\lambda; t_0, \bar{x}^\alpha)$ even at $\lambda = 0$;

(3) *Uniformity condition*: Let $\alpha = r$ and $\beta = \frac{\lambda}{r}$. The two conditions described above imply that $g = g(\alpha, \beta)$ is defined at $(0, 0)$, that for fixed $\beta = 0$ the metric g is smooth in α at least for $\alpha \neq 0$ and that for fixed $\alpha = 0$ the metric g is smooth in β at least for $\beta \neq 0$. The "uniformity condition" states that g is jointly smooth in (α, β) even at $(0, 0)$.

There are no known examples of such space-times and/or their initial data. The gluing described in the talk provides initial data which satisfy conditions similar to those of (1) and (2), but it is not clear that they will satisfy a condition analogous to that of (3).

References

[1] Corvino, J., *Scalar curvature deformation and a gluing construction for the Einstein constraint equations* Comm. Math. Phys. **214** (2000), no. 1, 137–189.

[2] Chruściel, P. T., Delay, E., *On mapping properties of the general relativistic constraints operator in weighted function spaces, with applications*, Mém. Soc. Math. Fr. (N.S.) No. 94 (2003)

[3] Isenberg, J., Mazzeo, R., Pollack, D., *On the topology of vacuum spacetimes* Ann. Henri Poincar **4** (2003), no. 2, 369–383.

[4] Chruściel, P. T., Isenberg, J., Pollack, D., *Initial data engineering*, Comm. Math. Phys. **257** (2005), no. 1, 29–42.

[5] Bartnik, Robert, *Regularity of variational maximal surface,* Acta Math. **161** (1988), no. 3-4, 145–181.

[6] Isenberg, J., Maxwell, D., Pollack, D. *A gluing construction for non-vacuum solutions of the Einstein-constraint equations*, Adv. Theor. Math. Phys. **9** (2005), no. 1, 129–172.

[7] Gralla, S., Wald, R., *A rigorous derivation of gravitational self-force*, http://arxiv.org/abs/0806.3293