

TRACEY PAYNE OPEN PROBLEMS

Questions about the Ricci flow for homogeneous spaces

- Suppose that a simply connected nilmanifold admits a left-invariant soliton metric g^* . Does an arbitrary left-invariant metric approach g^* under the projectivized Ricci flow?
- Let $\psi_t : \mathcal{N}_n \rightarrow \mathcal{N}_n$ denote the projectivized Ricci flow on the space of volume-normalized simply connected nilmanifolds of dimension n . How are the dynamical properties of ψ_t related to algebraic and geometric structures of the nilmanifolds? For example, what are invariant sets and monotone quantities for the flow?
- We have shown that, under the projectivized Ricci flow, a simply connected nilmanifold approaches a soliton nilmanifold. The nilpotent group N^* associated to the limiting nilmanifold may be different than the nilpotent group N for the original nilmanifold. Describe, in qualitative terms, how the algebraic structure of N^* is related to the algebraic structure of N . Is there a nice description of the function mapping N to N^* ?
- Describe the Ricci flow for other classes of Lie groups endowed with left-invariant inner products.
- Describe the Ricci flow for locally homogeneous spaces. For example, take a locally homogeneous metric on N/Γ , where N is nilpotent and Γ is a lattice in N .