

FUTURE WORK

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In the talk, we gave a construction for self-translating tridents under mean curvature flow, starting from the intersection of a grim reaper cylinder and a plane.

Self-translating surfaces.

Problem 1. *Can the procedure to construct translating tridents be used to desingularize the intersection of two grim reapers? Can it be generalized to any configuration of grim reapers and planes?*

In the case of two grim reapers, the construction should work as well. Because the setting enjoys less symmetry, solving the linear operator will be more delicate.

Completion of the construction of a self-shrinker.

Self-similar surfaces are surfaces that do not change shape but are homothetically contracted (self-shrinkers) or expanded (self-expanders) by the mean curvature flow. Self-shrinkers satisfy the equation

$$(1) \quad H + X \cdot \nu = 0,$$

where X is the position vector, H is the mean curvature function and ν is the normal vector given by $\mathbf{H} = H\nu$. The sign of H is chosen so that the mean curvature of a convex surface is positive.

These surfaces model the behavior of the mean curvature flow near singularities, under certain conditions [1]. Therefore, a better knowledge of self-similar surfaces will lead to a deeper understanding of the flow near singularities.

The second project consists in completing the construction of a self-similar surface by desingularizing the intersection of a sphere and a plane. From my previous results [3] [4], it is possible to perturb pieces of the approximate initial surface to obtain local solutions to (1), therefore the natural project is

Problem 2. *Find a method for gluing the solutions on the different pieces smoothly to obtain a global solution to the self-shrinker equation (1).*

This problem differs from Kapouleas' construction of complete embedded minimal surfaces [2] because Kapouleas was able to modify slightly the position and size of the catenoids and planes while here, the position and radius of the sphere and the position of the plane are fixed by (1).

Long term goal.

The strategy described above may also be used to desingularize the intersection of a cylinder and a plane, or a sphere and a cylinder to obtain a self-similar surface. More generally, my long term goal is to solve the problem below.

Problem 3. *Find a general method to desingularize the intersection of immersed rotationally symmetric surfaces and find the weakest conditions needed for such a method to work.*

REFERENCES

- [1] G. Huisken, *Asymptotic behavior for singularities of the mean curvature flow*, J. Differential Geometry 31 (1990) 285-299.
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- [3] X. H. Nguyen, *Construction of complete embedded self-similar surfaces under mean curvature flow. Part I*, preprint. arXiv:math/0610695
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