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Volume Estimates for Families of Cycles

If a 1-parameter family of curves sweeps across the unit disk, then one of the curves has length at least 2. We will discuss generalizations of this old result. By a result of Almgren, the space Z of unoriented 1-cycles in the unit disk has the (weak) homotopy type of infinite-dimensional real projective space. A 1-parameter family of 1-cycles sweeps out the disk mod 2 if the family represents a non-trivial class in $H_1(Z)$. We may then ask for a p -dimensional family of 1-cycles, representing a non-trivial homology class in $H_p(Z)$. We prove that such a family must contain a 1-cycle with length at least $cp^{1/2}$, and this estimate is sharp up to a constant factor. An example is provided by the family of algebraic cycles of degree at most d . Finally, we will touch on families of k -cycles in the unit n -ball.