PROBLEMS DISCUSSED AT THE SPRING 2006 PNGS

1. Problems on the Hamiltonian homeomorphism group, by Yong-Geun Oh

(1) Describe the closed set of length minimizing paths in terms of the geometry and dynamics of the Hamiltonian flows.
(2) Describe the set of topological Hamiltonians in $L_m^{(1,\infty)}([0,1] \times M, \mathbb{R})$.
(3) Study the structure of the flow of Hamiltonian homeomorphisms in terms of the C^0-Hamiltonian dynamical system or as the high dimensional generalization of area-preserving homeomorphisms with vanishing mass flow or zero mean rotation vector.
(4) Does the identity $[Sympeo_0, Sympeo_0] = Hameo$ hold? Is $Hameo$ simple?
(5) Is $Hameo(M, \omega)$ a Lie group?

2. Problems on Donaldson-Thomas and Gromov-Witten invariants of orbifolds and their crepant resolutions, by Jim Bryan

(1) Define Donaldson-Thomas invariants for orbifolds and formulate (1) an orbifold version of the Gromom-Witten/Donaldson-Thomas correspondence and (2) the Crepant Resolution Conjecture for Donaldson-Thomas invariants. The GW/DT correspondence (for non-orbifolds) was conjectured by Maulik-Nekrasov-Okounkov-Pandharipande and states that the reduced partition functions of the Donaldson-Thomas and Gromov-Witten invariants are equal after a certain change of variables. The Crepant Resolution Conjecture for Gromov-Witten theory has been formulated by Bryan-Graber and states that the Gromov-Witten partition functions of a Gorenstein orbifold and any crepant resolution are equal after a certain change of variables.
(2) Extend the definition of Donaldson-Thomas invariants to the symplectic category (they are currently only defined for algebraic manifolds). Thomas's original motivation and the conjectural correspondence with Gromov-Witten theory both suggest that Donaldson-Thomas theory should be defined in the symplectic category.
(3) Find a direct geometric characterization of the Gromov-Witten potential. In physics, the Gromov-Witten potential has intrinsic meaning but in mathematics, it is currently only formally defined – only the coefficients of the formal series have a geometric meaning.

3. Problems on global covergence of the Yamabe flow, by Simon Brendle

(1) Does the positive mass theorem hold in all dimensions and for all topologies? An affirmative answer to this question would imply the convergence of the Yamabe flow on any manifold.
(2) It would be interesting to gain a better understanding of the blow-up issues for the prescribed scalar curvature equation.

4. Problems on the Kahler-Ricci Flow and Monge-Ampere equation, by Gang Tian

Let (X, ω) be a Kahler manifold. Consider the normalized Kahler-Ricci flow

$$\frac{\partial \omega}{\partial t} = -(\mathrm{Ric}(\omega) - \lambda\omega)$$
$$\omega(0, \cdot) = \omega_0 \quad \lambda = -1, 0, 1$$

(1) ($\lambda = 1$) Conjecture: There exists $\varphi_t : X \to X$ such that $\varphi_t^* \omega(t, \cdot)$ converges in C^∞ to a Kahler-Ricci soliton (possibly on a different "manifold").

(2) By results of Tian-Zhang, Carsini-LaNave, Zhang, there exists solution $\omega(t, \cdot)$ for $t \in [0, T)$ where $T = \sup_t\{[\omega_t] > 0\}$, $\omega(t, \cdot) \to \omega(T, \cdot)$ (current) as $t \to T$ s.t. $\omega(T, \cdot)$ is smooth on $X - D$ and locally, $\omega(T, \cdot) = \partial\bar{\partial}\varphi$ for continuous φ. Speculations: Take X_T to be the metric completion of $X - D$ with respect to $\omega(T, \cdot)$. Then φ descends to X_T. Questions: 1) Does X_T exist as a variety? 2) How to define Ricci flow on X_T with only continuous initial data? 3) Why are there only finitley many T? 4) What is the limit metric at $t = \infty$. Some partial answers discussed.

5. Problems on semi-discrete curvature flows, by Ben Chow

(1) Combinatorial Yamabe flow $\frac{d}{dt} r_i = -K_i r_i$. When does this converge? Long time existence?