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What is Stokes' theorem – from algebraic foundations to geometric completions

Stokes' theorem $\int_{\partial M} \omega = \int_M d\omega$ is the favorite theorem of many mathematicians who find its generalized form beautiful, powerful, and concise. The question of its full extent led to the foundation of Bourbaki. Although the question was not fully answered in the Bourbaki series, its pursuit opened up new fields of mathematics. We will see how Stokes' theorem evolves from an infinitesimal algebraic formulation in a vector space to a full blown version over the completion of polyhedral chains in a normed space. The full power of the theory arises from an isomorphism of differential forms and cochains that preserves the exterior derivative operator d. The classical Cartan exterior calculus may be derived, as well as extensions of the exterior calculus to soap films, nonsmooth domains (such as graphs of L^1 functions) and discrete domains. Finally, we will see that if the dimension of the domain does not match the degree of the form, a Stokes' theorem still holds, but with three terms, rather than two.