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Does a random 3-manifold fiber over the circle?

If M is a hyperbolic 3-manifold, a conjecture of W. Thurston posits that M has a finite cover which fibers over the circle. This conjecture has proved fairly inscrutable, so here we try an oblique approach and address the question: How common is it for a 3-manifold to fiber over the circle? For a special class (those with tunnel number one) we will provide compelling evidence, both theoretical and experimental, that fibering is a very rare property. Indeed, in various precise senses it happens with probability 0.

The main ingredients needed are as follows. The first is work of Ken Brown from combinatorial group theory, which gives an algorithm to decide if a given tunnel number one 3-manifold fibers over the circle. Then, techniques of Agol, Hass, and W. Thurston can be adapted to calculate very efficiently by using the recursive structure of embedded curves in a genus 2 surface. This algorithm works in the context of splitting train-tracks/interval exchanges, which are basically analogues of continued fraction expansions of real numbers. To analyze the algorithm, we generalize work of Kerchoff to understand the dynamics of splitting sequences of complete genus 2 interval exchanges. In particular, we show this system is normal. Finally, recent work of Mirzakhani as plays an important role. Combining all of these with a "magic splitting sequence" proves the main theorem. (This is joint work with Dylan Thurston, Columbia University.)