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Complex Ray Singer torsion and a few applications

(joint work with S. Haller)

The Ray Singer torsion is a positive real number, associated to a smooth compact odd dimensional manifold and a unitary representation of its fundamental group. It is a topological invariant and can be calculated from the spectra of the Laplace-Beltrami operators (on forms with values in the representation), associated to a Riemannian metric. They are self adjoint nonnegative operators, hence their spectrum is real and nonnegative.

We will show that the Ray Singer torsion is the absolute value of a complex number, the complex Ray-Singer torsion, which can be associated to any manifold and any representation of the fundamental group (at the expense of an additional topological structure, a co-Euler structure).

The complex Ray Singer torsion is defined using non positive laplacians whose spectrum consists of complex numbers. The phase (argument) of the complex Ray Singer torsion is an interesting topological invariant analogous to, but different from, the eta invariant (when defined).

As an application we provide a rational function on the space of complex representations of a fixed rank. Many of the familiar rational functions in topology can be recovered from this function. A Cheeger Muller theorem which compares this Ray Singer torsion with the Milnor Turaev torsion also holds.