PACIFIC NORTHWEST GEOMETRY SEMINAR

OPEN PROBLEMS, SPRING 2004

Brian White

1. What is the infimum among all minimal Möbius strips in $\mathbf{R}^3$ of the total curvature of the boundary curve? (By my talk the number is $> 3\pi$ and $< 4\pi$.)

2. -among all oriented minimal surfaces of genus > 1? (By example, this is at most 4π. It is conjectured to be 4π.)

3. -among all nonorientable minimal surfaces other than Möbius strips?

4. Can a pair of convex curves in parallel planes bound a smooth connected minimal surface other than an annulus?

5. Conjecture: A smooth curve embedded in $\mathbf{R}^3$ with total curvature at most 4π bounds, in addition to the unique minimal disk (Nitsche's theorem), either 0, 1, or 2 minimal Möbius strips, and no other (classical) minimal surfaces. If there are two Möbius strips, one is strictly stable and the other has index of instablitiy 1. (If there is just one, it is stable, but just barely.) (Meeks and I proved a related result for any pair of convex curves in parallel planes.)

6. If you have a small lawn and want to replace your lawn mower with a goat, how do you keep the goat from eating the grass right down to the roots?

Ian Agol: Tameness of hyperbolic 3-manifolds

1. Given a pinched negatively curved 3-manifold with finitely generated fundamental group, is the convex core biLipschitz to the convex core of a hyperbolic 3-manifold?

2. Are 3-manifolds with finitely generated fundamental group and a CAT(0) metric tame? What about for delta-hyperbolic metrics?

3. Is the boundary of the convex core of a pinched negatively curved 3-manifold with pinching constants between $b < a < 0$ CAT(a) and Alexandrov curved $> b$? Is the boundary $C^{1,1}$ everywhere except along an embedded curve?

Michael Wolf: An Embedded Genus One Helicoid

1. Are all genus-one helicoids embedded? Numerical evidence of Bobenko indicates not, but a proof is lacking. Is the embedded genus-one helicoid unique? Are the embedded screw-motion invariant simply periodic surfaces with genus-one quotients unique?

2. Prove the existence of an embedded minimal surface of genus g with a single end asymptotic to a helicoid. Can one continue the Traizet-Weber germ to a geometric limit of a genus g helicoid? Is this germ unique? More vaguely, explain the occurrence of Hermite polynomials in the Traizet-Weber theory – how do we see those polynomials in terms of a flow of two-ended minimal surfaces in a family of degenerating three-manifolds of topology $S^1 \times E^2$?

3. Are there embedded minimal surfaces in space which are isolated in the moduli space of minimal surfaces, i.e. are not part of a family of minimal surfaces that degenerates?

Ben Chow: Ricci Flow and Fukaya Theory in Dimension Three

(1) Let $(M^2, g(t))$ be a complete solution to the Ricci flow on a surface with curvature bounded at each time on an ancient time interval $(-\infty, \omega)$, where $-\infty < \omega \leq \infty$. Applying the maximum principle to the evolution equation for the scalar curvature R, if $g(t)$ is not flat for all t, then $R[g(t)] > 0$ for all time. Assume the latter is true.
   - (a) If M^2 is compact, must $g(t)$ be rotationally symmetric? Is $g(t)$ in fact isometric to a constant multiple of the Rosenau solution (see Chapter 2 of [1] for a description of the Rosenau solution)?
   - (b) If M^2 is noncompact, must $g(t)$ be isometric to a constant multiple of the cigar soliton ($\mathbb{R}^2$ with the metric $\frac{dx^2+dy^2}{1+x^2+y^2}$)?

(2) Let $(M^3, g(t))$ be a complete solution to the Ricci flow on a 3-manifold with curvature bounded at each time on an ancient time interval $(-\infty, \omega)$, where $-\infty < \omega \leq \infty$. By the Hamilton-Ivey estimate, $g(t)$ has nonnegative sectional curvature. Suppose this solution is κ-noncollapsed and has positive sectional curvature.
   - (a) (Perelman) If M^3 is noncompact, must $g(t)$ be isometric to a constant multiple of the Bryant soliton?
   - (b) If M^3 is compact and diffeomorphic to S^3 or $\mathbb{R}P^3$, must $g(t)$ be rotationally symmetric (or its $\mathbb{Z}_2$ quotient)?

(c) If M^3 is compact and not diffeomorphic to S^3 or $\mathbb{R}P^3$, must $g(t)$ have constant sectional curvature?

References

[1] Chow, Bennett; Knopf, Dan, *The Ricci flow: An introduction,* Mathematical Surveys and Monographs, AMS, Providence, RI, 2004.