Math 582E **Complex Manifolds** **Winter 2022**

SYLLABUS

Lectures: MWF 1:30–2:20
Jan 3 – Jan 7: Zoom (link available on Canvas)
Jan 10 – Mar 11: Smith 309

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Course website: www.math.washington.edu/~lee/Courses/582-2022

Course Description: Complex manifolds are defined exactly the same way as smooth manifolds, except the local coordinate charts are required to take their values in $\mathbb{C}^n$ and to overlap holomorphically. But the similarity ends there. For example, on a compact complex manifold, the only global holomorphic functions are the constants, and the space of holomorphic sections of a holomorphic vector bundle is always finite-dimensional. While every smooth manifold can be embedded in some Euclidean space, only certain complex manifolds can be embedded in $\mathbb{C}^n$ or in complex projective space. There is a deep interplay between differential geometry and complex analysis, especially for Kähler manifolds, the ones on which the metric structure and the complex structure play together nicely.

Complex manifolds have deep and beautiful applications in many areas of mathematics. Here are a few examples:

> Riemann surfaces (one-dimensional complex manifolds) are essential for understanding global properties of holomorphic functions in one complex variable.
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> Complex surfaces (two-dimensional complex manifolds) play a central role in attempts to classify 4-dimensional smooth manifolds.
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> Complex manifolds defined by algebraic equations are among the central objects of interest in algebraic geometry, and the study of their differential geometry has contributed important advances in algebraic geometry.
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> Calabi–Yau manifolds are complex manifolds that play a crucial role in string theory.

I hope to cover the following topics:

- Definition and examples of complex manifolds
- Almost complex structures and integrability
- Holomorphic vector bundles
- Line bundles and hypersurfaces
- Sheaves and cohomology
- Hermitian connections
- Hermitian and Kähler metrics
- Hodge theory
- Brief overview of Kähler-Einstein metrics and Calabi-Yau manifolds
- The Kodaira embedding theorem

Prerequisites: Topology, smooth manifolds, Riemannian geometry, vector bundles, and undergraduate complex analysis.

Books:

The official text for this course will be an online draft textbook that I'll be writing throughout the quarter. I'll probably provide an updated version approximately each week or so.

In addition, you're invited to consult the following books, all of which are all available from the Math Research Library (either in book form or downloadable through the UW Libraries website).

> Werner Ballmann, *Lectures on Kähler Manifolds*, European Mathematical Society, 2006.
> Phillip Griffiths and Joseph Harris, *Principles of Algebraic Geometry*, Wiley, 1994.
> Daniel Huybrechts, *Complex Geometry: An Introduction*, Springer, 2005.
> Andrei Moroianu, *Lectures on Kähler Geometry*, Cambridge, 2007.
> Raymond O. Wells, Jr., *Differential Analysis on Complex Manifolds*, 3rd ed., Springer, 2008.
> Fangyang Zheng, *Complex differential geometry*, AMS, 2000.

Lectures:
I'll lecture via Zoom (during the first week) or in person (later) at our scheduled class times. On Zoom, I'll usually show up five or ten minutes early for Q & A before lecture starts.

During a lecture, feel free to ask questions. If I'm in the middle of explaining something, raise your hand (or use the Zoom "raise hand" button) to get my attention. Any time I pause, just speak up.

Homework:
I'll assign problem sets at irregular intervals, usually every couple of weeks. You should do all the assigned reading, and try to figure out how to do all the *Exercises* included in the text, whether assigned or not. The problems listed as "to write up and hand in" are to be submitted via Canvas for grading. In order to count, they need to be uploaded no later than midnight on the due date.

When you write up your solutions, please follow these guidelines:

> **Citing results:** You may freely cite theorems, propositions, corollaries, lemmas, and exercises from earlier in the textbook. But (unless I announce otherwise) the result of a *problem* can only be used if it has been previously assigned, or if you give its solution. You may also use anything from my books *Introduction to Topological Manifolds*, *Introduction to Smooth Manifolds*, or *Introduction to Riemannian Manifolds*, including the results of problems and exercises, unless they are substantially identical to what you're being asked to prove. If you look up and use something proved in any other book or on the internet, please explain what you found and where you found it, and write up a proof in your own words of any result that you need to use to solve a homework problem. Do not look up specific solutions to the assigned problems, and do not copy specific language that anyone else has written.
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> **Collaboration:** I strongly encourage you to work with other students on the homework. Discussing problems and ideas with your classmates is one of the best ways to absorb new ideas. But when writing up solutions to hand in, you must *write your own solutions in your own words.*
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> **Writing it up:** Always start by writing the problem number (from the book), and stating what you're going to prove. You don't have to copy the whole problem statement verbatim;

often it's better just to state the theorem that you're being asked to prove. Don't be stingy with white space: start each problem on a new page, and leave one-inch margins on all sides of your pages. Arrange your solutions in numerical order, just as they appear on the assignment page. Problems that are out of order might not get credit. I've posted a link on Canvas to an essay I wrote titled "Some Remarks on Writing Mathematical Proofs"; I strongly urge you to read that and follow its advice.

Typesetting vs. handwriting: If you are comfortable doing so, I encourage you to submit computer-typeset assignments. I highly recommend LaTeX, since that is the de facto standard in mathematics; but any typesetting program will do. I've posted some helpful typesetting links on Canvas. I'm also happy to accept handwritten assignments, as long as they are neat and legible (see below).

Legibility: If you write by hand, write your answers neatly and legibly, not too small, with as few erasures or crossouts as possible. Be sure to distinguish clearly between similar symbols. Unless mathematical ideas spring fully and impeccably realized from your pen, your first draft is not acceptable.

Submission: When you've finished the assignment, create a PDF copy of the assignment and upload it to Canvas. If you need to scan handwritten pages and don't have a regular scanner, try the smartphone app called Genius Scan, which is available free for iPhone and Android.

Religious Accommodations:
Washington state law requires that UW develop a policy for accommodation of student absences or significant hardship due to reasons of faith or conscience, or for organized religious activities. The UW's policy, including more information about how to request an accommodation, is available at *Religious Accommodations Policy* (registrar.washington.edu/staffandfaculty/religious-accommodations-policy/). Accommodations must be requested within the first two weeks of this course using the *Religious Accommodations Request form* (registrar.washington.edu/students/religious-accommodations-request/).

Disability Accommodations:
It is the policy and practice of the University of Washington to create inclusive and accessible learning environments consistent with federal and state law. If you have already established accommodations with Disability Resources for Students (DRS), please activate your accommodations via myDRS so we can discuss how they will be implemented in this course. If you have not yet established services through DRS, but have a temporary health condition or permanent disability that requires accommodations, contact DRS at disability.uw.edu.

Grading:
Your grade will be based on the required homework problems; there are no exams. The cutoff for a 4.0 will be approximately 80%, and the cutoff for a 3.0 approximately 50%.

If you wish, you may register for this course on an S/NS basis. Be sure to tell me if that's what you're doing. In this case, if you attend (or listen to) lectures regularly and hand in essentially correct written solutions to at least three homework problems taken from three different assignments, I'll record your grade as a 2.7, which will be converted by the registrar to S (satisfactory).