Math 548 | **Geometric Structures** | **Winter 1997**

Cohomology, Bundles, and Connections

COURSE DESCRIPTION

Lectures: MWF 1:30–2:20
Padelford C-401

Instructor: Jack Lee
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Office Hours: Mon. & Wed. 11:30–12:30 or by appointment

Web site: `http://www.math.washington.edu/~lee/Courses/548`
(or from the Math Department home page, follow the links
Courses → **Math 548**)

Books:

There will be no textbook for this course. Attached is a list of suggested references that you can consult if you wish; these books will be placed on reserve in the Math Research Library.

Prerequisite:

Math 547 (Riemannian geometry). If you haven't taken Math 547 recently and would like to brush up on your Riemannian geometry, you can download a copy of last quarter's Math 547 lecture notes on the web. From the Math Department home page, follow the links **Courses** → **Archived course web pages** → **Math 547** → **Lecture notes**.

Some acquaintance with homology theory and/or partial differential equations would be helpful, but I will not assume any prior knowledge of these subjects.

Homework and grades:

I will assign homework problems on an irregular basis. Your grade will be based on how many of these you do correctly: roughly speaking, getting 80% of the assigned problems correct will get you a 4.0, and 50% a 3.0.

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SYLLABUS

Cohomology:	Simplicial complexes Simplicial homology and cohomology Sheaves Sheaf cohomology The deRham Theorem
Bundles:	Vector bundles: definitions and examples Operations on vector bundles Constructing new bundles from old Fiber bundles Structure groups G-structures Principal bundles and associated bundles Spin structures Classifying spaces Introduction to characteristic classes
Connections:	Connections in vector bundles Connections in principal bundles Curvature Flat bundles Chern-Weil theory Higher Chern and Pontryagin classes The Euler class The Chern-Gauss-Bonnet Theorem

If time permits, I will also try to give a *very brief* description of the basic ideas of gauge theory (including Donaldson and Seiberg-Witten invariants).

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SUGGESTED READING

The following books should be on 24-hour reserve in the Math Research Library.

1. R. Bott and L. W. Tu, *Differential Forms in Algebraic Topology*, New York, Springer-Verlag, 1982. This excellent book covers many of the same topics we will cover, albeit with a very different emphasis due to the algebraic-topological orientation of its authors.

2. S.-S. Chern, *Complex manifolds without potential theory*, New York, Springer-Verlag, 1979. Although this is primarily a book about complex manifold theory, its appendix gives an excellent general treatment of characteristic classes of real and complex vector bundles.

3. R. C. Gunning, *Lectures on Riemann Surfaces*, Princeton, Princeton University Press, 1966. Although this is a book about Riemann surfaces, §§2–3 give a treatment of sheaves and sheaf cohomology very similar to the one I will give.

4. M. W. Hirsch, *Differential topology*, New York, Springer-Verlag, 1976. Good treatment of classifying spaces for vector bundles.

5. S. Kobayashi, *Transformation groups in differential geometry*, New York, Springer-Verlag, 1972. The main reference for the theory of G-structures.

6. S. Kobayashi and K. Nomizu, *Foundations of Differential Geometry*, New York, Interscience Publishers, 1969. This is a fairly comprehensive reference for differential geometry, with a strong emphasis on the principal bundle approach, and a thorough treatment of connections and characteristic classes. It's hard to read if you don't already have a basic understanding of the material, but it's a good reference.

7. H. B. Lawson and M.-L. Michelsohn, *Spin geometry*, Princeton, Princeton University Press, 1989. A comprehensive introduction to spin algebra, spin bundles, Dirac operators, and their applications in geometry and topology.

8. J. W. Milnor and J. D. Stasheff, *Characteristic classes*, Princeton, Princeton University Press, 1974. This is the standard reference for the topological construction of characteristic classes. It also has an appendix explaining the Chern/Weil construction (using connections).

9. J. Munkres, *Elements of Algebraic Topology*, Reading, Addison-Wesley, 1984. An excellent elementary introduction to simplicial homology and cohomology.

10. M. Spivak, *A comprehensive introduction to differential geometry*, Berkeley, Publish or Perish, 1979. This is a classic. It's overly verbose, and not as complete as one might like, but many people find it a fun read. The basic theory of vector bundles is laid out in Volume 1, principal bundles and connections are treated in various ways in Volume 2, and characteristic classes are treated in Volume 5.

11. N. Steenrod, *The topology of fibre bundles*, Princeton, Princeton University Press, 1957. This was one of the first books to develop the general topological theory of fiber bundles. It's pretty old-fashioned, but quite readable, and the concreteness of the presentation has much to recommend it.

12. F. Warner, *Foundations of Differentiable Manifolds and Lie Groups*, New York, Springer, 1983. This classic text contains, in addition to all the basic material on differentiable manifolds, a complete treatment of sheaf cohomology and the deRham theorem. The proof is quite a bit different from the one I will give, however.