Math 547 **Geometric Structures** **Winter 1996**

SYLLABUS

Lectures: MWF 1:30–2:20
Communications B006

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(or from the Math Department home page, follow the links
Current Class Information → Math 547)

Textbooks:

There will be no required textbook for this course. Instead, I will write my own lecture notes and hand them out as they are available. For further information, you are encouraged to consult the following sources (all on reserve in the Mathematics Research Library):

- W. M. Boothby, *An Introduction to Differentiable Manifolds and Riemannian Geometry, 2nd ed.*, Academic Press, Orlando, 1986.
- M. DoCarmo, *Riemannian Geometry*, Birkhäuser, Boston, 1992.
- F. Morgan, *Riemannian Geometry: A Beginner's Guide*, Jones and Bartlett, Boston, 1993.
- M. Spivak, *A Comprehensive Introduction to Differential Geometry, Vols. I–V*, Publish or Perish, Berkeley, 1979.
- J. Cheeger and D. G. Ebin, *Comparison Theorems in Riemannian Geometry*, North-Holland, Amsterdam, 1975.
- S. Kobayashi and K. Nomizu, *Foundations of Differential Geometry, Vols. I–II*, Interscience, New York, 1963.

General description:

This course introduces the most important concepts and tools of Riemannian geometry. Central to the subject is the concept of *curvature*, which we will study from a variety of points of view. After introducing the definitions and studying local properties of curvature, we will proceed to some of the "big theorems" relating curvature to global topological properties.

Prerequisites:

Topology and Geometry of Manifolds (three quarters); specifically, the material in Chapters 1–6 of the Boothby text listed above.

Homework:

The homework problems will be divided into three groups:

1. *Practice problems:* These are "exercises for the reader" integrated into the text of the lecture notes. They may include any of the following types of problems: straightforward generalizations of things done in the lectures; elementary proofs that "fill in the gaps" from lectures; background material that you should have already seen in an earlier course; computations of simple but illuminating examples; intermediate results that will be used in class or in other problems. You are expected to do *all* of the problems in this category, but they will not be collected or graded. I recommend that you write up the solutions for your notes; but at the very least, you should read each problem carefully and make sure you know exactly how to do it.

2. *Required problems:* These problems are the heart of the course, and are to be written up and handed in for a grade. They will be assigned approximately once every two weeks, due two weeks later. They will typically include some proofs and some computations of significant examples.

3. *Optional problems:* There will usually be a few optional problems in addition to the required problems on each assignment. Some of them will carry the ideas from the course farther, or lead you through proofs that we don't have time to cover. Some of them are just particularly difficult examples. If you have time, and if you really want to understand the subject, I encourage you to try as many of these as you can, and turn in those you complete successfully. (See also *Grading*, below.) You may turn in optional problems after an assignment's due date, but only if you get permission from me on or before the due date.

The practice and required homework problems represent the minimum work I think you need to do to master this material sufficiently well to pursue a Ph.D. in any field of pure mathematics that requires a working knowledge of Riemannian geometry. If you plan to do research in differential geometry, you should try to do all the optional problems as well.

You are encouraged to work together on the homework problems (it's usually the best and fastest way to learn). However, when you write up your solutions to hand in, *you must write your own solutions in your own words.*

Grading:

Your grade will be based on the required and optional homework problems. Roughly speaking, getting 90% of the required problems correct will get you a 4.0, 70% will get you a 3.0, and grades in between will be linearly interpolated. Optional problems handed in will be counted for extra credit if they are mostly correct. A grade lower than 3.0 means you are not performing at the level expected of a graduate student.

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COURSE OUTLINE

1. **Introduction**
    - What is curvature?
    - Review of Riemannian metrics, tensors, and their basic properties
2. **Connections and geodesics**
    - Linear connections
    - Definition of geodesics; existence, uniqueness
    - Covariant derivatives along curves; parallel translation
3. **The Levi-Civita connection**
    - Symmetry and the torsion tensor
    - The Fundamental Lemma of Riemannian Geometry
    - The exponential map, normal neighborhoods and normal coordinates
    - Examples of Riemannian manifolds and their geodesics
4. **Minimizing properties of geodesics**
    - Riemannian manifolds as metric spaces
    - The first variation formula for arc length
    - The Gauss Lemma and the proof that geodesics are locally minimizing
    - Completeness; the Hopf-Rinow theorem
5. **Curvature**
    - The Riemann curvature operator
    - Flat metrics
    - The curvature tensor and its symmetries
6. **Riemannian submanifolds**
    - The shape operator and the second fundamental form
    - Principal curvatures; Gaussian and mean curvatures
    - The Gauss equation and the *Theorema Egregium*
    - Sectional curvature
    - Manifolds of constant curvature
7. **The Gauss-Bonnet Theorem**
    - The Rotation-Angle Theorem
    - The Gauss-Bonnet Formula and The Gauss-Bonnet Theorem
    - Higher-dimensional generalizations
8. **The second variation and Jacobi fields**
    - The second variation formula for arc length
    - The Jacobi equation and Jacobi fields
    - Conjugate points
    - Global minimizing properties of geodesics
9. **Curvature and topology**
    - The Cartan-Hadamard Theorem
    - Bonnet's Theorem
    - Myers's Theorem