Mathematics 547/548: Geometric Structures

John M. Lee

MWF 1:30–2:20

Math 547 (Winter Quarter)
Riemannian Manifolds, Connections, Curvature, and Geodesics

This course introduces the concepts and tools of Riemannian geometry. Central to the subject is *curvature*, which we will study from a variety of points of view. After studying local properties of curvature, we will proceed to some of the "big theorems" relating curvature to global topology.

Prerequisites: *Topology and Geometry of Manifolds* (Math 544/5/6).

- Directional derivatives of vector fields; the idea of a connection.
- The Levi-Civita connection on a Riemannian manifold; the "Fundamental Lemma of Riemannian Geometry."
- Definition of geodesics; local minimizing properties of geodesics; normal neighborhoods, the exponential map, and normal coordinates.
- Riemannian manifolds as metric spaces; geodesic and metric completeness.
- Various notions of curvature and their geometric interpretations.
- Geometry of Riemannian submanifolds; the second fundamental form.
- The Gauss-Bonnet theorem: *If M is a compact oriented Riemannian 2-manifold with Gaussian curvature K, then $\int_M K\,dA = 2\pi\chi(M)$, where $\chi(M)$ is the Euler characteristic of M, a topological invariant.*
- Manifolds of constant curvature.
- Jacobi fields and conjugate points; comparison theorems.
- The Cartan-Hadamard Theorem: *If M is a complete Riemannian manifold with non-positive sectional curvature, the universal cover of M is diffeomorphic to $\mathbf{R}^n$.*
- The Bonnet-Meyers Theorem: *If M is a complete Riemannian manifold with Ricci curvature bounded below by $\kappa > 0$, then M is compact, with finite fundamental group and diameter at most equal to that of the sphere with constant Ricci curvature κ.*

Math 548 (Spring Quarter)
Special topics

During the spring quarter, this course will cover one or more advanced special topics in differential geometry. The exact topic or topics to be chosen will depend on the interests of the students who enroll, but I tentatively plan to cover an introduction to complex manifolds. The course will be open to students who have already taken Math 548 in previous years.

Prerequisites: Math 547. If the topic is Complex Manifolds, then *Complex Variables* will also be prerequisite (at least Math 534/5).

If you are interested in taking Math 548 but are not enrolled in Math 547, please contact the instructor sometime during the Fall quarter (`lee@math.washington.edu`, 543-1735).