Math 545/546 **Topology and Geometry of Manifolds** **Winter/Spring 2000**

SYLLABUS

Lectures:	MWF 1:30–2:20 Padelford C-36
Instructor:	Jack Lee Padelford C-546 543-1735 *lee@math.washington.edu* Office Hours: Mon & Wed 10:30–11:30 or by appointment
Teaching Assistant:	Eduardo Chappa-Leiva Padelford C-17 *chappa@math.washington.edu* Office Hours: Tue & Thu 12:00–1:00 or by appointment
Course Web site:	*http://www.math.washington.edu/~lee/Courses/545-2000* (or from the Math Department home page, **Selected Course Web Pages → Math 545**)
Required text:	J. M. Lee, *Introduction to Smooth Manifolds: Notes for Math 545 and 546*, to be distributed in class.
Supplementary texts:	J. M. Lee, *Introduction to Topological Manifolds: Notes for Math 544*, available at Professional Copy & Print, 4200 Univ. Way NE. W. M. Boothby, *An Introduction to Differentiable Manifolds and Riemannian Geometry, 2nd ed.*, Academic Press, Orlando, 1986 (on reserve). F. W. Warner, *Foundations of Differentiable Manifolds and Lie Groups*, Springer–Verlag, New York, 1983 (on reserve).

General description:

This course continues the study of manifolds begun in Math 544. For these two quarters, the subject will be *smooth* or *differentiable manifolds*, which are manifolds on which derivatives of functions and maps make sense. We will study the basic flora and fauna that live on them: submanifolds, tangent vectors, vector fields, flows, Riemannian metrics and their simple properties, tensor fields, differential forms, orientations. The basic theory and examples of Lie groups (which are groups that are also smooth manifolds) will be woven throughout the course.

Prerequisites:

Topology. All the material covered in Math 544.

Linear Algebra. Abstract vector spaces, dimension, bases, linear maps, change of basis, inner products, bilinear forms, linear functionals, dual spaces. References: any abstract linear algebra text, such as *Linear Algebra: An Introductory Approach* by C. Curtis or *Finite-Dimensional Vector Spaces* by P. Halmos.

Real Analysis. Calculus of mappings from $\mathbb{R}^m$ to $\mathbb{R}^n$: partial derivatives, the chain rule, the total derivative as a linear approximation, multiple integrals, the change of variables formula. Vector analysis: divergence, gradient, curl, and the theorems of Green, Gauss, and Stokes. Ordinary Differential Equations: existence and uniqueness theorems, techniques for solving first-order systems at the level of Math 307 and 309. Metric spaces: complete metric spaces, function spaces, uniform convergence, differentiation and integration of sequences and series. References: *Principles of Mathematical Analysis* by W. Rudin; any advanced calculus textbook; any ODE textbook.

Homework:

A homework assignment will be given out once a week, due one week later. A typical homework assignment will consist of three parts:

1. *Reading:* Typically, you will be given one chapter to read each week. I will expect you to actually read it, and to do all the exercises that are included in the text! Most of these exercises will not be collected and graded, but nevertheless you need to do them in order to understand everything that's being discussed.

2. *Required Problems:* Each week a few problems will be assigned for you to write up and hand in. These problems are the heart of the course.

3. *Optional problems:* Some of these problems carry the ideas from the course further, or lead you through proofs that we don't have time to cover. Some of them are just particularly difficult examples. If you have time, and if you really want to understand the subject, I encourage you to try as many of these as you can. I'll be more than happy to discuss them with you outside of class. (They won't officially count in your numerical grade, but if your grade is on a borderline they can influence which way it goes.)

I encourage you to form study groups to work together on the homework problems (it's usually the best and fastest way to learn). However, when you write up your solutions to hand in, *you must write your own solutions in your own words.*

Lectures:

In my lectures, I'll try to explain the definitions and meanings of all the important concepts we'll use in the course; to present many of the significant proofs in detail; and to give examples and intuitive explanations of "what it all means." But it is impossible to understand this much material deeply after only three hours a week. You shouldn't expect, therefore, to absorb everything I say in lecture as it happens. To supplement the lectures, you need to (1) read the chapters before coming to lecture; and (2) study your lecture notes carefully after class, working through any details or exercises that are still unclear.

Outside reading:

A unique feature of manifold theory is the wide variety of approaches, alternate definitions, and notation systems that have developed over the years. So that we may have a common language for this course, I have chosen a specific set of conventions for my notes, which are by and large the ones most commonly used in the mathematical community. However, it is important that you expose yourself to other points of view.

During each quarter, you'll be required to do a little outside reading on one of the topics we cover in class. Pick a topic or set of topics representing one or two weeks' worth of lectures, find another book that covers those topics, and carefully read the corresponding sections of that book, filling in any omitted details and doing some relevant exercises. Aim to cover about a chapter. You may choose one of the books on the reading list I will give out, or you may choose another book. In either case, you must clear your selection with me in advance, and report back to me orally on what you read (see "Individual meetings", below).

Individual meetings:

You are required to schedule and attend two 15-minute meetings with me during each quarter: once before the end of the fourth week, to discuss your choice of outside reading material, and once before the end of the ninth week, to report back on what you've read.

Exams:

At the end of each quarter there will be a take-home final exam. It will be somewhere between a long homework assignment and a manifolds prelim, covering material from the entire quarter.

Grading:

Your grade will be based 2/3 on the required homework problems and 1/3 on the exam. I will not decide on the exact scale until the end of the quarter, but when I've taught this course in past years an average of about 90% yielded a 4.0 and 70% a 3.0, with scores in between linearly interpolated. The minimum grade for receiving graduate credit is 3.0.

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COURSE OUTLINE

Winter Quarter

Smooth manifolds: Charts and atlases; smooth structures on manifolds; examples of smooth manifolds; smooth functions and maps; manifolds with boundary.

Tangent vectors: Tangent vectors in $\mathbb{R}^n$; tangent vectors on a manifold; coordinate vectors; push-forwards; computations in coordinates; the tangent bundle as a smooth manifold.

Submanifolds: Immersions, embeddings, and submersions; submanifolds; level sets and parametrizations; the inverse and implicit function theorems; the rank theorem.

Embedding and approximation: Bump functions and partitions of unity; the Whitney embedding theorem; the Whitney approximation theorem.

Introduction to Lie groups: Definition of a Lie group; Lie subgroups; group actions on manifolds; equivariant maps; smooth covering maps.

Vector fields and flows: Vector fields; existence, uniqueness, and smoothness theorems for ODEs; integral curves and flows; Lie derivatives; Lie brackets; normal forms for commuting vector fields.

Tangent distributions and foliations: Tangent distributions; involutivity and integrability; the Frobenius theorem; foliations.

Spring Quarter

Lie groups and Lie algebras: Review of Lie groups; left-invariant vector fields; the Lie algebra of a Lie group; the classical groups; one-parameter subgroups; the exponential map; homogeneous spaces; the correspondence between Lie groups and Lie algebras.

Tensors: Covectors and covector fields; the cotangent bundle; tensors and tensor fields; Riemannian metrics.

Differential forms: Alternating tensors; differential forms; wedge products; orientation; exterior derivatives; integration of differential forms; the Poincaré lemma.

Stokes's theorem: Stokes's theorem; deRham cohomology; singular homology and cohomology; the deRham theorem; examples and computations.