Math 545 **Topology and Geometry of Manifolds** **Winter 2000**

Assignment #6

Due 2/18/2000

I. Required problems.

1. Suppose $M \subset N$ is a closed embedded submanifold and $f \in C^\infty(M)$. (This means f is smooth as a function on M, not as a function on a closed subset of N.) Show that f is the restriction of a smooth function on N. Find a counterexample to this result if the hypothesis that M is closed is omitted.
2. Let $M \subset \mathbb{R}^m$ be an embedded submanifold, let U be a tubular neighborhood of M, and let $r \colon U \longrightarrow M$ be the retraction defined in Proposition 4.24. Show that U can be chosen small enough that for each $x \in U$, $r(x)$ is the point in M closest to x. [Hint: first show that each point $x \in U$ has a closest point $y \in M$, and this point satisfies $(x - y) \perp T_y M$.]
3. If $M \subset \mathbb{R}^m$ is an embedded compact submanifold and $\varepsilon > 0$, let M_ε be the set of points in $\mathbb{R}^m$ whose distance from M is less than ε. Show that for sufficiently small ε, ∂M_ε is a compact embedded submanifold of $\mathbb{R}^m$, and $\overline{M}_\varepsilon$ is a smooth manifold with boundary.

II. Optional problems.

4. Let $M \subset \mathbb{R}^m$ be an embedded submanifold, and let NM be its normal bundle Show NM is a vector bundle with projection $\pi \colon NM \longrightarrow M$. [Hint: To construct a local trivialization, let (y^i) be slice coordinates and apply the Gram-Schmidt algorithm to the vectors $\partial/\partial y^i$.]