Math 545 **Topology and Geometry of Manifolds** **Winter 2000**

Assignment #4

Due 2/2/2000

I. Required problems.

1. If an n-manifold is diffeomorphic to an m-manifold, prove that $n = m$.
2. (a) Show that a proper injective immersion is an embedding.
   (b) Show that an embedded submanifold is closed if and only if the embedding is proper.
3. Suppose M is an n-manifold and $y^1, \ldots, y^k$ are smooth real-valued functions on M. Define a map $F \colon M \longrightarrow \mathbb{R}^k$ by $F(x) = (y^1(x), \ldots, y^k(x))$. Suppose F has rank r at a point $p \in M$.
   (a) If $r = k = n$, show that $(y^1, \ldots, y^n)$ are coordinates for M in some neighborhood of p.
   (b) If $r = k < n$, show there are functions $y^{k+1}, \ldots, y^n$ such that $(y^1, \ldots, y^n)$ are coordinates for M in some neighborhood of p.
   (c) If $r = n < k$, show that there are indices $i_1, \ldots, i_n$ such that $(y^{i_1}, \ldots, y^{i_n})$ are coordinates for M in some neighborhood of p.
4. Exercise 3.1 in the notes.