**Math 544** **Topology and Geometry of Manifolds** **Autumn 1997**

# SYLLABUS

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| **Lectures:** | MWF 1:30–2:20<br>Padelford C-36 |
| **Instructor:** | John M. Lee<br>Padelford C-546<br>543-1735<br>`lee@math.washington.edu`<br>Office Hours: Wed. & Fri. 10:30–11:30 or by appointment |
| **Course Web Site:** | `http://www.math.washington.edu/~lee/Courses/544`, or from the Math Department home page, **Courses** → **Math 544**. I will post the lecture notes and homework assignments here as they are available. |
| **Texts:** | [L] John M. Lee, *Introduction to Topological Manifolds*, notes for this course. I will hand them out a chapter at a time as they are available; they will also be posted on the Web.<br>[S] Allan J. Sieradski, *An Introduction to Topology and Homotopy*, PWS-Kent, Boston, 1992 (*recommended*). |

## Prerequisites

*Set Theory:* Basic facts of "naive set theory": functions, equivalence relations, countability, order relations, the well-ordering theorem. References: Chapter 1 of [S]; *Naive Set Theory* by P. R. Halmos.

*Algebra:* Elementary group theory: homomorphisms, isomorphisms, subgroups, normal subgroups, cosets, quotient groups. Reference: *Topics in Algebra* by N. Herstein.

*Analysis:* Open and closed sets, continuous maps, convergence, metric spaces. Reference: *Principles of Mathematical Analysis* by W. Rudin.

If you aren't familiar with one or two of these topics, you should spend some time reading the reference books listed above. If, however, you find a lot of items on the list unfamiliar, you should consider taking a lower-level course that covers some of these prerequisites.

**Note:** *the winter and spring quarters of this course, Math 545-546, will also require a solid knowledge of advanced calculus and linear algebra. See Professor Judith Arms for specific prerequisites.*

## General description

*Manifolds* are arbitrary-dimensional generalizations of curves and surfaces; they show up in nearly every field of pure mathematics, from number theory to partial differential equations, as well as in many applied fields such as mechanics, general relativity, and elementary particle theory. In the fall quarter we will concentrate on the *topology* of manifolds, i.e., properties that are invariant under continuous deformations. The main tool here is the fundamental group, which is a group associated to every manifold in such a way that topologically equivalent manifolds have isomorphic groups. In later quarters you will see *smooth* or *differentiable manifolds*, on which derivatives of functions and maps make sense.

## Course goals

This course is designed to prepare you to work toward a Ph.D. in pure mathematics, so there is a strong emphasis on mastering the use of technical tools and methods of proof. However, we will continually strive to keep our feet on the ground by looking at concrete examples and doing lots of computations.

If you satisfactorily complete all three quarters of this course, you should have mastered the basic concepts and tools of manifold theory sufficiently well to use them when they arise in other fields of study; you should have the background necessary for the Manifolds preliminary exam; and you should be well prepared to continue your study by taking any second-year course that deals with manifolds, such as *Geometric Structures*, *Algebraic Topology*, or *Lie Groups and Lie Algebras*.

However, we can't cover everything, and some important topics are left out. This first-year course will not cover in any detail the geometry of curves and surfaces, Riemannian geometry (curvature, the Gauss-Bonnet Theorem), or piecewise linear topology.

## Homework

The lecture notes will contain two kinds of questions for the reader: *exercises*, which are integrated into the text, and *problems*, grouped at the ends of the chapters.

The *exercises* include the following types of problems: straightforward generalizations of things we've done in class; elementary proofs that "fill in the gaps" from lectures; computations of simple but illuminating examples; intermediate results that will be used in class or in other problems. I expect you to do all of the exercises. I recommend that you write up the solutions for your notes; but at the very least, you should read each exercise carefully and make sure you know exactly how to solve it.

The *problems* at the ends of the chapters are the heart of the course. Approximately three or four problems will be assigned each week for you to write up and hand in a week later for grading. They will typically include some proofs and some computations of significant examples. If you have time, and if you really want to understand the subject, I encourage you to try also to do as many of the unassigned problems as you can. I'll be more than happy to discuss them with you outside of class.

The homework will sometimes take a considerable amount of time and effort. Because this is a 5 credit course, and the University's expectation is that each credit represents an average of 3 hours per week of student effort, the workload is designed under the assumption that you will spend a total of 15 hours a week on this course *on the average*, including class meetings, studying, homework, and exams. This means some of you might have to spend considerably more than 15 hours in some weeks. The reward for all this work, of course, is that by the end of the year you will have mastered some of the most subtle and powerful tools in mathematics.

I strongly encourage you to work together in study groups on the homework problems—it's usually the best and fastest way to learn. However, when you write up your solutions to hand in, you must write your own solutions in your own words.

## Lectures

In my lectures, I will try to explain the definitions and meanings of all the important concepts we'll use in the course; to present many of the significant proofs in detail; and to give examples and intuitive explanations of "what it all means." But it is simply impossible to convey this much material, in three hours a week, in a way that will make you understand it deeply. You should not expect, therefore, to understand or absorb everything I say in lecture as it happens. While I welcome questions from the class during lecture, I expect you will have to take good class notes, study your class notes and the printed lecture notes carefully, and do the exercises and problems before you will feel comfortable with the material. The real learning in this course takes place when you do the problems, and you should look at the lectures and notes primarily as supplements to the homework.

## Final Exam

At the end of the quarter there will be a take-home final exam covering material from the entire quarter. It will be handed out during the last week of class, and will be due on Wednesday, December 17.

## Grading

Your grade will be based 2/3 on the required homework problems and 1/3 on the exam. Roughly speaking, getting 90% of the homework and exam problems correct will get you a 4.0, 70% will get you a 3.0, and scores in between will be linearly interpolated. A grade lower than 3.0 means you are not performing at the level expected of a graduate student.

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# COURSE OUTLINE

*Each topic below (except the first) will take approximately a week.*

1. **Why study manifolds?**
   Examples of manifolds and how they arise in various branches of mathematics.
2. **Some point-set topology**
   Metric spaces and topological spaces; continuous functions; Hausdorff spaces; subspaces, product spaces, and quotient spaces; countability axioms.
3. **Manifolds**
   Definition of topological manifolds; connectedness and compactness.
4. **Surfaces**
   Connected sums; cutting and pasting; presentations of surfaces; classification theorem for compact surfaces.
5. **The fundamental group**
   Homotopy; the fundamental group; action of continuous maps on fundamental groups; topological and homotopy invariance.
6. **The circle**
   The fundamental group of the circle; path lifting lemmas; contractibility.
7. **Some group theory**
   Free products of groups; free groups; presentation of groups by generators and relations.
8. **Fundamental groups of surfaces**
   The Seifert-Van Kampen theorem; computation of the fundamental groups of compact surfaces.
9. **Covering spaces**
   Definition and examples of covering spaces; liftings of paths, homotopies, and maps; relationship between the fundamental group and covering spaces; solution of the lifting problem.
10. **Classification of coverings**
    Isomorphisms and automorphisms of coverings; existence of a universal covering space; the classification theorem.