Corrections to Introduction to Topological Manifolds

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- **Page 5, last sentence of second paragraph:** The dates are wrong. The parenthesized phrase should read: "(for 5-manifolds and higher by Steve Smale in 1960 [Sma60], and for 4-manifolds by Michael Freedman in 1982 [Fre82])."
- **Page 7, next to last paragraph:** *special unitary group* should be italicized.
- **Page 11, third paragraph:** Delete the last sentence (the one starting "Finally, we discuss ... ").
- **Page 12, fourth paragraph:** In the definition of a metric, d should map $M \times M \to \mathbb{R}$ (not $M \times M \to M$).
- **Page 15, fourth paragraph from bottom:** "converges to y" should read "converges to q."
- **Page 23, first line:** should read "X is said to be *first countable* if there exists a countable neighborhood basis at each point." [Insert "countable."]
- **Page 23, end of first paragraph:** Delete the sentence "Other than for purposes of illustration, we will not use first countability in these notes."
- **Page 23, last paragraph:** "locally Euclidean" should read "locally Euclidean of dimension n."
- **Page 25, Problem 2-7:** Insert the following sentence somewhere in the problem: "Assume X has at least two elements."
- **Page 25:** Delete Problem 2-9.
- **Page 29, display at top of page:** Each occurrence of $W \underset{\text{open}}{\subset} A$ should be changed to $W \underset{\text{open}}{\subset} X$ (3 times).
- **Pages 30–31, Example 3.5:** Change the definition of F to

  $$F(\varphi, \theta) = ((2 + \cos 2\pi\varphi)\cos 2\pi\theta, (2 + \cos 2\pi\varphi)\sin 2\pi\theta, \sin 2\pi\varphi).$$

  Also, change the first line of page 31 to read $F(\varphi + k, \theta + l) = F(\varphi, \theta)$, and change the first displayed equations to read

  $$\varphi = \frac{1}{2\pi}\tan^{-1}\frac{z}{\sqrt{x^2+y^2}-2} + k; \qquad \theta = \frac{1}{2\pi}\tan^{-1}\frac{y}{x} + l;$$

  $$\varphi = \frac{1}{2\pi}\cot^{-1}\frac{\sqrt{x^2+y^2}-2}{z} + k; \qquad \theta = \frac{1}{2\pi}\cot^{-1}\frac{x}{y} + l.$$

  (This is so that F will map the unit square $I \times I$ onto D, which is used later on page 49.)

- **Page 32, statement of Theorem 3.8:** (3.1) should read "if and only if the following composite map from B to X is continuous:".
- **Page 38, Example 3.17:** first sentence should read "Consider the map $q\colon \mathbb{R}^{n+1} - \{0\} \to \mathbb{S}^n$ ... ." In the second sentence, R^{n+1} should be $\mathbb{R}^{n+1}$.
- **Page 46, proof of Theorem 4.3:** Second sentence should read " ... or to be open subsets of Y whose union contains $f(X)$."
- **Page 47, second paragraph in proof of Proposition 4.5:** Second sentence should read " ... disjoint open sets U_p containing q and V_p containing p ... ."
- **Page 52, Exercise 4.3:** The hint should read " ... which set contains $\sup(U \cap [a,b])$?"
- **Page 56, paragraph before Lemma 4.22:** First sentence should read "A space can be connected but not locally connected, as is, for example, the closed topologist's sine curve (see Problem 4-5); ... ."
- **Page 60, displayed equation:** Delete the factors of 2 from the arguments of sin and cos.
- **Page 63, first full paragraph:** Change "insure" to "ensure" in the sentence beginning "Shrinking U_1 and U_2 if necessary, ..."
- **Page 67, last sentence:** should read " ... edges of the k-gon in clockwise order ... " (not counterclockwise).
- **Page 70, last line:** should read " ... has the presentation $\langle a_1, \dots, a_n \mid a_1 a_1 \cdots a_n a_n \rangle$; ... ."
- **Page 71, Lemma 5.11(f):** display should read

  $$\langle S, a \mid W_1 a a^{-1} W_2 \rangle \mapsto \langle S \mid W_1 W_2 \rangle.$$

  (Insert missing a in the the first presentation.)
- **Page 74, Step 1, second sentence:** should read "Since M is connected, if there are two or more faces, ..."
- **Page 75, first paragraph:** Insert the following sentence at the very end of the first paragraph: "We may also have created some new adjacent opposing pairs. These can be eliminated by repeating Step 2; since that step does no reflections, it cannot introduce any new aligned pairs."
- **Page 75, last paragraph:** The sentence after the display should read "In the original presentation, there were two vertices labeled v, one labeled w, and two labeled x, ..."
- **Page 76, first line:** should read " ... , so perform Step 2 again ... ."
- **Page 78, Problem 5.2(d):** Delete the last comma in the presentation, so the last two words are replaced by the single word $j^{-1}n^{-1}o^{-1}$.
- **Page 85, statement and proof of Theorem 6.9:** Change $[g]^{-1}$ to $[g^{-1}]$ in both the statement and the proof.

- **Page 86, paragraph after Exercise 6.4:** second sentence should read "$\mathbb{R}^2 - \{0\}$" instead of "$\mathbb{R}^n - \{0\}$."
- **Page 104, proof of Theorem 7.1, second paragraph, third line:** should read "$\widetilde{f}(1) = n$" instead of "$\widetilde{f}(1) = 2\pi n$."
- **Page 106, sentence after the statement of Lemma 7.6:** should read "Any number ε satisfying the conditions in this lemma is called a *Lebesgue number* for the cover."
- **Page 107, first line after top display:** should read " ... to a map $\widetilde{H}\colon I \times I \to \mathbb{R}$ ... ."
- **Page 118, last sentence of Example 8.4:** should read " ... i_1 or j_m may be zero, ..."
- **Page 122, line 12 from bottom:** "for every $g \in K$" should read "for every $g \in G$ ... ."
- **Page 133, proof of Proposition 9.7, third line:** should read "Since V is homeomorphic to $\mathbb{R}^n$ ... ."
- **Page 134, Figure 9.1:** Change the label $\widetilde{g}$ to g in the upper part of the figure.
- **Page 136, third paragraph:** should read " ... where b is an expression for $j_*[\widetilde{c}] \in \pi_1(\widetilde{V}, \widetilde{q})$ ... ." [Add a tilde on the last q.]
- **Page 140, Figure 9.2:** The label in the small rectangle inside the square should be R_{ij}, not S_{ij}.
- **Page 140, first line of text:** should read "Thus the path obtained by restricting H to the bottom edge ... ."
- **Page 150, Example 10.3:** Change "the preceding example" to "Example 10.1."
- **Page 155, sentence before second display:** should read "Since the lift of $p \circ g$ is g, ..." [Delete the tilde.]
- **Page 161, Corollary 10.24:** second line should read " ... then for any $q \in X$ and $\widetilde{q} \in p^{-1}(q)$, ..." [Delete two tildes.]
- **Page 163, Figure 10.1:** Replace φ by φ_1.
- **Page 165, third paragraph, third line:** should read "every conjugacy class" (not "every conjugate class").