Math 544Topology and Geometry of ManifoldsAssignment #8, Supplementary Problems

Required Problems

A. CLASSIFICATION OF COMPACT 2-MANIFOLDS WITH BOUNDARY: Show that every compact 2manifold with boundary is homeomorphic to a compact 2-manifold with finitely many regular disks removed.

Optional Problems

- B. This problem shows how the classification theorem can be extended to some noncompact 2-manifolds. A manifold M is said to have *finite topological type* if there is a subset $K \subseteq M$ such that
 - (a) K is a compact manifold with boundary;
 - (b) $\overline{M \setminus K}$ is homeomorphic to $\partial K \times [0, \infty)$.

(A connected sum of infinitely many tori is an example of a manifold that does *not* have finite topological type.) Prove that a noncompact 2-manifold has finite topological type if and only if it is homeomorphic to a compact 2-manifold with finitely many points removed.