Math 544/5/6 **Topology and Geometry of Manifolds** **2001–2002**

SYLLABUS

Lectures: MWF 1:30–2:20
Padelford C-36

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Course Web site: *http://www.math.washington.edu/~lee/Courses/544*
(or from the Math Department home page,
Selected Course Web Pages → Math 544)

Text for 544: [ITM] J. M. Lee, *Introduction to Topological Manifolds*, Springer-Verlag, Graduate Texts in Mathematics #202, 2000.

Text for 545/546: [ISM] J. M. Lee, *Introduction to Smooth Manifolds*, Springer-Verlag, to be published in 2002. (Partial preprints will be made available if the published version is not released in time.)

General description:

Manifolds are arbitrary-dimensional generalizations of curves and surfaces—spaces that locally look like Euclidean space but globally may not, just as the sphere locally looks like the plane but is globally very different. They are the basic subject matter of differential geometry, but also play a role in many other branches of pure and applied mathematics. In the fall quarter we will concentrate on the *topology* of manifolds, i.e., properties that are invariant under continuous deformations. The main goals are to understand the fundamental group, covering spaces, and the classification of compact surfaces. The winter and spring quarters will be devoted to the study of *smooth manifolds*, on which derivatives of functions and maps make sense. We will study the basic flora and fauna that live on them: tangent vectors and covectors, submanifolds, vector and tensor fields, Riemannian metrics, differential forms, orientations, flows. We will also examine the basic examples of manifolds with geometric structures, such as Lie groups, Riemannian manifolds, and symplectic manifolds.

Prerequisites:

For Fall quarter

- *Set Theory*: Operations on sets, functions, equivalence and order relations, number systems and cardinality, the axiom of choice. References: *Principles of Mathematical Analysis* by Rudin, Chapter 1; *Naive Set Theory* by Halmos; Appendix to [ITM].
- *Analysis*: Metric spaces; convergence and continuity; open and closed sets; interior, exterior, and boundary; compactness. Reference: *Principles of Mathematical Analysis* by Rudin, Chapters 2,3,4; Appendix to [ITM].
- *Algebra*: Elementary group theory, homomorphisms, isomorphisms, subgroups, normal subgroups, permutation groups, cyclic groups, cosets, quotient groups. Reference: *Abstract Algebra: An Introduction* by Hungerford, Chapter 7; Appendix to [ITM].

For Winter and Spring quarters

- *Topology*: All the material covered in Math 544. Reference: [ITM].
- *Linear algebra*: Abstract vector spaces, subspaces, bases, dimension, matrices, determinants, change of basis formulas, linear maps, kernel and image, norms and inner products, orthonormal bases. Reference: any abstract linear algebra text, such as *Linear Algebra* by Friedberg, Insel, and Spence; Appendix to [ISM].
- *Multivariable calculus*: Partial derivatives; the total derivative as a linear approximation; Taylor's formula in several variables; multiple integrals and the change of variables formula; gradient, divergence, and curl; the theorems of Green, Gauss, and Stokes; uniform convergence. References: *Basic Multivariable Calculus* by Marsden, Tromba, and Weinstein; *Principles of Mathematical Analysis* by Rudin, Chapters 5,6,7; Appendix to [ISM].
- *Differential equations*: Basic facts about existence and uniqueness of solutions to ODEs; elementary techniques for solving first-order equations and systems at the level of Math 307 and 309. Reference: *Elementary Differential Equations and Boundary Value Problems* by Boyce and DiPrima.

Homework:

A homework assignment will be given out once a week, due one week later. A typical homework assignment will consist of the following:

1. *Reading:* Typically, you will be given approximately one chapter to read each week. I will expect you to read through the chapter quickly before the relevant lectures, and then to reread it carefully after the lecture.

2. *Reading Report:* About once a week, I'll ask you to send me a brief report about your reading in the textbook. These reports need not be long or detailed (one or two paragraphs should suffice), but each report should include at least the following:

    - What you think are the one or two most important concepts in the section you read.
    - One or two important questions about the reading that you'd like to see answered in class.

3. *Exercises:* Scattered throughout the textbooks are various *exercises* (as distinct from the *problems* at the ends of the chapters). I expect you to do (or at least figure out how to do) *all of them* as you read. Most of these are relatively routine, and will not be collected or graded, but nevertheless understanding them is important to the continuity of the text and the lectures.

4. *Written Exercises and Problems:* Each week a few exercises and/or problems will be assigned for you to write up and hand in. These problems are the heart of the course.

5. *Optional Problems:* Most weeks I will list some problems as optional. Some of these will carry the ideas from the course further, or lead you through proofs that we don't have time to cover. Some of them are just particularly difficult examples. If you have time, and if you really want to understand the subject, I encourage you to try as many of these as you can. I'll be more than happy to discuss them with you outside of class. (They won't officially count in your numerical grade, but if your grade is on a borderline they can influence which way it goes.)

I encourage you to form study groups and work together on the homework problems (it's usually the best and fastest way to learn). However, when you write up your solutions to hand in, *you must write your own solutions in your own words.*

Outside Reading:

A unique feature of manifold theory is the wide variety of approaches, alternative definitions, and notation systems that have developed over the years. In order to have a common language, one must choose a particular set of conventions; the ones I have chosen for my books are by and large the conventions most commonly used in the mathematical community. However, it is important that you expose yourself to other points of view. During each quarter, I'll hand out a list of suggested books for outside reading. You are required to pick a chapter from our text, find another book that covers the same material, and read the corresponding section(s) of that book, filling in any omitted details and doing some relevant exercises. (If you pick a book that's not on my list, please clear it with me in advance.) No later than the last day of classes, send me an e-mail report on your outside reading, a few paragraphs long.

Exams:

At the end of each quarter there will be a take-home final exam. It will be somewhere between a long homework assignment and a manifolds prelim, covering material from the entire quarter.

Grading:

Your grade will be based 2/3 on the required homework problems and 1/3 on the exam. I will not decide on the exact scale until the end of the quarter, but when I've taught this course in past years an average of about 90% yielded a 4.0 and 70% a 3.0, with scores in between linearly interpolated. The minimum grade that can be counted toward a degree in mathematics is 3.0.