

**Part I:**

Each of the statements below is an implication. For each statement, do all of the following:

- Identify the hypothesis and the conclusion.
  - Write the converse.
  - Write the inverse.
  - Write the contrapositive.
1. If  $P$ ,  $Q$ , and  $R$  lie on  $\ell$ , then they are collinear.
  2. If  $\ell$  is a line, then it contains at least two distinct points.
  3. A quadrilateral is a parallelogram if it is a rectangle.
  4. For a triangle to be isosceles, it is necessary that it have two equal angles.
  5.  $x$  is divisible by 4 only if it is even.
  6. If  $2x + 1 = 5$ , then  $x = 2$  or  $x = 3$ .
  7. If the  $10^{100}$ th decimal digit of  $\pi$  is 3, then  $\sqrt{5} = 2$ .

**OVER**

**Part II:**

8. Venema, page 42, Exercises 3.1, 3.2.
9. Write the negations of each of the three incidence axioms.
10. Write the negation of each of the following statements.
  - (a) If  $P$ ,  $Q$ , and  $R$  all lie on  $\ell$ , then they are collinear.
  - (b)  $P$  lies on  $\ell$  or it lies on  $m$ .
  - (c) For any three points  $P$ ,  $Q$ , and  $R$ , if they are collinear, then there is another point  $S$  that is not equal to  $P$ ,  $Q$ , or  $R$ .
  - (d) For every line  $\ell$ , if  $\ell$  contains three distinct points, then it has points in common with three distinct lines.
  - (e) There exists a line  $\ell$  such that for every point  $P$ ,  $P$  lies on  $\ell$ .
  - (f) There exists a point  $P$  that does not lie on any line.
11. Below is the outline of a proof of Theorem 3.6.2. Fill in the blanks with appropriate reasons.

**Theorem 3.6.2.** *If  $\ell$  is any line, then there exists at least one point  $P$  such that  $P$  does not lie on  $\ell$ .*

**Proof:**

| Statement   | Reason |
|---|--------|
| 1. Let $\ell$ be a line.  | _____  |
| 2. Let $P$ , $Q$ , and $R$ be three noncollinear points.                  | _____  |
| 3. $P$ , $Q$ , and $R$ do not all lie on any one line.                    | _____  |
| 4. At least one of the points $P$ , $Q$ , or $R$ does not lie on $\ell$ . | _____  |
| 5. There is a point that does not lie on $\ell$ .                         | _____  |

**Part III:**

12. Write proofs in two-column format for Venema's Theorems 3.6.3 and 3.6.4 on page 41.