Math 444 **Geometry for Teachers** **Winter 2008**

Handout #5: Reading Guide for Venema's Section 5.7

This section contains a diverse collection of theorems, unified by a common idea: They are all "obvious-seeming" results that follow from the first four postulates, but that often require some work to prove. The main thrust of this section, therefore, is to justify many of the kinds of conclusions that Euclid drew from his diagrams.

Here is my suggestion for how to approach the chapter. First, read the main results, without worrying about their proofs, and make sure you understand what they say. They are:

- **Theorem 5.7.1 (Betweenness Theorem for Points).**
- **Corollary 5.7.2** (You could call this **The Whole Segment is Greater Than the Part**).
- **Corollary 5.7.3** (Of any three distinct collinear points, exactly one is between the other two).
- **Theorem 5.7.5 (Existence and Uniqueness of Midpoints).**
- **Theorem 5.7.10** (I call this **Betweenness vs. Betweenness**).
- **Theorem 5.7.12 (Betweenness Theorem for Rays).**
- **Theorem 5.7.14 (Existence and Uniqueness of Angle Bisectors).**
- **Theorem 5.7.15 (The Crossbar Theorem).**
- **Theorem 5.7.19 (Linear Pair Theorem).**
- **Theorem 5.7.23 (Existence and Uniqueness of Perpendicular Bisectors).**
- **Theorem 5.7.25 (Vertical Angles Theorem).**

Next, go back to the beginning and read the chapter in sections.

Betweenness for Points: Reread through Theorem 5.7.5 (assigned last week).

The Crossbar Theorem: Next, tackle the crossbar theorem and its proof. (Skip Corollary 5.7.8, Theorem 5.7.10, Lemma 5.7.11, Theorem 5.7.12, and Theorem 5.7.14 for now.) There are three preliminary results leading up to the proof of the crossbar theorem:

- **Theorem 5.7.6:** This is not particularly useful in itself; it's mainly used to prove the next corollary (the Y-theorem).
- **Corollary 5.7.7** (I call this **The Y-Theorem**): This is one of those "obvious" results that will often come in handy later.
- **Corollary 5.7.9 (The Z-Theorem):** Another useful "obvious" result, which is an easy consequence of the Y-theorem.
- To the above I added one in class that I called **The X-Theorem:** If ℓ and m are lines that intersect at a point A, and $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are opposite rays on m, then every point on $\overrightarrow{AB}$ (except A) lies on the opposite side of ℓ from every point on $\overrightarrow{AB}$ (except A).

Then read the proof of the crossbar theorem itself. After you've read Venema's proof, read Eric Heye's proof (which I handed out in class, and is available on the class website under Handouts), and decide which you prefer.

Betweenness for Rays: Now go back and read the parts you skipped (Corollary 5.7.8, Theorem 5.7.10, Lemma 5.7.11, Theorem 5.7.12, and Theorem 5.7.14), and also look at Theorem 5.7.16.

Linear Pairs and Vertical Angles: Read from the middle of Page 79 through Theorem 5.7.25. (Skip the discussion of the Continuity axiom.)