Math 443 **Differential Geometry** **Spring 2013**

Assignment #8

Due 6/7/13 (for practice only; not to be handed in)

Reading:

- [AT] Sections 6.2 and 6.3
- [AT] Guided Problems 6.1, 6.2, 6.3

Written Problems:

(1) [AT] Exercise 6.5.

(2) Verify the Gauss–Bonnet theorem for the torus of revolution generated by the circle whose equation is $(y-2)^2 + z^2 = 1$, by computing the value of each term separately. (See also Exercise 4.41.)

(3) Suppose $R \subseteq \mathbb{R}^3$ is a regular domain that is diffeomorphic to a closed disk. Suppose further that its boundary curve is the image of a closed geodesic. Prove that the Gaussian curvature of R must be positive somewhere.

(4) Suppose $R \subseteq \mathbb{R}^3$ is a regular domain that is diffeomorphic to the cylinder $S^1 \times [0,1] \subseteq \mathbb{R}^3$. Suppose further that both boundary curves are images of closed geodesics. If the Gaussian curvature of R is not identically zero, prove that it attains both positive and negative values.

(5) Let S be the paraboloid defined by $z = x^2 + y^2$, and for each $r > 0$, let S_r be the portion of S where $z \leq r$. Verify the Gauss–Bonnet formula for S_r by computing each term separately.