Math 443 **Differential Geometry** **Spring 2011**

Assignment #7: Due 5/20/11

Reading:

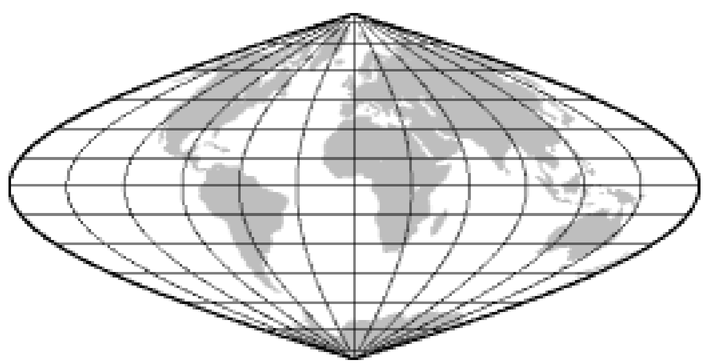
- Bär, Sections 4.10, 4.11 (again).
- Bär, Section 3.4 (again).
- Bär, p. 182 in Section 4.5 (again).
- [Optional] Do Carmo, Section 2.6 (pp. 103–108).

Written Assignment:

A. The map below is created using ***sinusoidal projection***, which is the parametrization $F\colon U \to S^2$ defined by

$$F(u,v) = \left(\cos\left(\frac{u}{\cos v}\right)\cos v,\ \sin\left(\frac{u}{\cos v}\right)\cos v,\ \sin v\right),$$

defined on the domain $U = \{(u,v) : |v| < \pi/2,\ |u| < \pi|\cos v|\}$. Compute the matrix of the first fundamental form for this parametrization, and determine if it is conformal, area-preserving, or neither.



B. The Poincaré half-plane model of the hyperbolic plane is the surface $HP = \{(r,s) : s > 0\}$ with the Riemannian metric

$$\big(g_{ij}(r,s)\big) = \begin{pmatrix} \frac{1}{s^2} & 0 \\ 0 & \frac{1}{s^2} \end{pmatrix}.$$

(a) For arbitrary $r_0 \in \mathbb{R}$, find a parametrization of the vertical line $r_0 =$ constant that is unit-speed with respect to this metric, and prove that it is a geodesic.

(b) For any real number a, prove that the map $\tau_a\colon HP \to HP$ given by $\tau_a(r,s) = (r+a,s)$ is an isometry.

(c) For any positive real number b, prove that the map $\delta_b\colon HP \to HP$ given by $\delta_b(r,s) = (br,bs)$ is an isometry.

C. Bär, Exercise 4.37 (p. 222). [Note that for some reason, Bär labels his coordinates (v,u) in HP, so u labels the vertical axis, not the horizontal one. Don't take the phrase "vertices at infinity" too seriously: the region Δ is just the set of all points $(v,u) \in HP$ such that $|v| \le 1$ and $u^2 + v^2 \ge 1$.]

D. Let S^2 be the unit sphere in $\mathbb{R}^3$, with the first fundamental form as its Riemannian metric, and with the orientation determined by the outward unit normal. For fixed $\varphi \in (-\pi/2, \pi/2)$, let $c\colon \mathbb{R} \to S^2$ be the latitude circle parametrized by $c(t) = (\cos\varphi\cos t, \cos\varphi\sin t, \sin\varphi)$. Calculate the geodesic curvature of c.