Math 443 | **Differential Geometry** | **Spring 2011**

Assignment #2: Due 4/15/11

Reading:

- Sections 4.3, 4.4, 4.5.

Written Assignment:

A. Prove that a composition of isometries is an isometry, and a composition of local isometries is a local isometry.

B. Exercise 4.11 (p. 159)

C. Exercise 4.16 (p. 171)

D. Let $c\colon I \to \mathbb{R}^2$ be a parametrized unit-speed plane curve given by $c(t) = (r(t), s(t))$, and let S be the surface of revolution generated by c, with parametrization given by

$$F(t, \varphi) = (r(t)\cos\varphi, r(t)\sin\varphi, s(t)).$$

Recall that last quarter (Assignment #7), you computed that the matrix of the first fundamental form in this parametrization is given by

$$(g_{ij}) = \begin{pmatrix} 1 & 0 \\ 0 & r(t)^2 \end{pmatrix},$$

and that the Gaussian curvature at a point $p = F(t, \phi)$ is

$$K(p) = -\ddot{r}(t)/r(t).$$

(a) Using the formula above for (g_{ij}), compute the eight Christoffel symbols and the sixteen components of the Riemann curvature tensor.

(b) For an arbitrary point $p = F(u)$, find an orthonormal basis for T_pS and verify the Theorema Egregium by showing that the Gauss curvature is also given by the formula of Theorem 4.3.8.

E. Exercise 3.28 (p. 142) gives a family of parametrizations F_α for $\alpha \in \mathbb{R}$ whose images S_α are all minimal surfaces. (See Assignment #9 from last quarter.)

(a) Prove that F_0 is a global parametrization of the helicoid of Example 3.8.14.

(b) Prove that if $F_{\pi/2}$ is restricted to a suitably small open set, it is a local parametrization of the catenoid of Example 3.8.13.

(c) For the parametrizations F_0 and $F_{\pi/2}$, compute the coefficients of the first fundamental form.

(d) Show that the catenoid is locally isometric to the helicoid.

(e) Are the catenoid and helicoid globally isometric? Explain.