I. Reading:
  • do Carmo, Section 4-6 (pp. 283–294).

II. Practice problems:
  1. do Carmo, Section 4-5 (pp. 282–283) #1, 4, 8

III. Required problems:
  1. do Carmo, Section 4-5 (pp. 282–283) #6.
  2. do Carmo, Section 4-5 (pp. 282–283) #7.
  3. Let \( X(u, v) = (\sin u \cos v, \sin u \sin v, \cos u) \) be the spherical coordinate parametrization for \( S^2 \), and let \( R = X([0, \pi/4] \times [\pi/4, \pi/2]) \). Verify the Gauss-Bonnet formula for \( R \) by computing each term separately.
  4. Let \( f, g : \mathbb{R} \to \mathbb{R} \) be smooth functions with \( f(v) > 0 \) for all \( v \), and let \( S \) be the surface of revolution in \( \mathbb{R}^3 \) generated by the curve \((y, z) = (f(v), g(v))\). Suppose that \( f'(a) = f'(b) = 0 \) for some \( a < b \in \mathbb{R} \), and let \( R \subset S \) be the regular region obtained by restricting \( f \) and \( g \) to the closed interval \([a, b] \). Show that \( \int_R K \, d\sigma = 0 \).