Math 442 **Differential Geometry** **Winter 2013**

Syllabus

Lectures:	MWF 12:30-1:20 Loew 205
Instructor:	John M. Lee Office: Padelford C-546 Phone: 206-543-1735 Email: *johnmlee@uw.edu*
Course Web site:	*www.math.washington.edu/~lee/Courses/442-2013* (or from the Math Department home page, **Current Course Web Pages → Math 442A**)
Textbook:	*Curves and Surfaces*, by M. Abate & F. Tovena

General description:

Broadly speaking, "differential geometry" is the use of tools of calculus to study problems in geometry. More specifically, in this course, we will apply tools of multivariable and vector calculus to study geometric properties of curves and surfaces in 3-dimensional space. Whereas topology, the subject matter of Math 441, is concerned with properties that are preserved by homeomorphisms (continuous bijections with continuous inverses), geometry is concerned with properties that are preserved by distance-preserving homeomorphisms. Such properties include distances (of course) and angles, as well as other familiar non-topological properties such as lengths of curves, areas, and volumes, all of which require calculus to compute in general. And the most important geometric property of all is one that cannot even be defined without calculus—curvature.

Specifically, in 442 and 443 we will cover most or all of the following topics: the theory of curves in the plane and in 3-space, the extrinsic local theory of surfaces in 3-space (looking at a surface from outside), the intrinsic local theory of surfaces (looking at a surface from inside), geodesics, curvature, the Gauss–Bonnet theorem, and the global theory of surfaces.

Because of the focus on calculus, the flavor of the course will be very different from that of 441. It will be in some ways less abstract than 441, but because we will introduce quite a few new technical tools, you will need to learn at least as many new definitions, concepts, and techniques as in 441. And the course will probably tax your understanding of linear algebra and multivariable calculus to the limit. The payoff is that you should end up having mastered the tools needed to delve into one of the most beautiful and physically relevant branches of modern mathematics.

Prerequisites:

The official prerequisite is grades of 2.0 or higher in Math 308, 326, 328, and 441. However, you'll have a much better chance of succeeding in this course if you've received grades of 3.0 or higher in the prerequisite 300-level courses, and at least a 2.5 in 441. In addition, the most important prerequisite is a genuine interest in abstract mathematics, or at least a mind open to the possibility. Intellectual curiosity and an ability to express mathematics clearly in writing are also important.

Specific prerequisite material includes the following. If two or three items on the list are unfamiliar, you are probably still OK as long as you're willing to spend some extra time reviewing. If there are

half a dozen unfamiliar items on the list, you're not ready for this course. Of course many people will be rusty on various topics. Some of these topics will be reviewed briefly in class as they arise, but be prepared to do some review on your own.

- Vector calculus: partial derivatives, the chain rule, dot products, cross products, tangent lines, tangent planes, line integrals, surface integrals, gradients, vector fields, the divergence theorem.
- Linear algebra: vector spaces, bases and dimension, linear maps and their representation by matrices, rank of a matrix, determinants, matrix algebra, eigenvectors and eigenvalues.
- Analysis in several variables: functions of several variables and their differentials; the inverse and implicit function theorems; the change of variables theorem for double and triple integrals.
- Topology: open and closed sets, boundaries, limit points, closures, continuous maps, homeomorphisms, connected sets, compact sets.

Homework:

A homework assignment will be given out each week, due a week later. Plan to spend a lot of time on homework—six hours or more outside of class during most weeks.

A typical homework assignment will consist of a reading assignment and some written problems to be handed in for a grade. Here are some other important instructions regarding homework:

- **Collaboration:** I strongly encourage you to work with other students on the homework. Discussing problems and ideas with your classmates is one of the best ways to learn the material. You'll get the most benefit from working with others if you make a good faith effort to solve the problems on your own first; but once you've thought about them for a while, I don't mind if you ask each other for explanations of how to do the problems. Here's the most important requirement: when writing up solutions to hand in, *you must write your own solutions in your own words.* It is not acceptable to copy from someone else's solution, or from another book, or from something you find on the Internet. If I receive homework papers that are verbatim or near-verbatim copies of each other, both papers will receive zero credit.
- **Stating theorems:** Begin each homework exercise with a statement of what you're doing. It's not necessary, however, to copy the problem statement verbatim. If the exercise asks you to "prove" or "show" something (these words mean the same thing), then please *state what you're proving in the form of a theorem.* Thus, for example, if the exercise says "prove that parametrized straight lines have zero curvature," you could start your solution with "**Theorem:** Every parametrized straight line has zero curvature." (If you write "prove that parametrized straight lines have zero curvature," who is that command directed to?)
- **Citing results:** You may freely cite theorems and assigned exercises from earlier in the book, theorems that I've proved in class, and theorems from Math 441. If you do use a previous result, be sure you identify it clearly: by giving its name, or its theorem number, or stating it correctly. And be sure its hypotheses are satisfied in the situation in which you want to apply it!
- **Assembly:** Arrange your solutions in numerical order, just as they appear on the assignment sheet, with each problem starting on a new page. Problems that are out of order might not get credit. Please staple the pages of each assignment together.
- **Identification:** Make sure the first page of each homework packet is clearly labeled with your name and the assignment number.

- **Typesetting vs. handwriting:** If you are comfortable doing so, I encourage you to submit computer-typeset assignments. I highly recommend LaTeX, since that is the de facto standard in mathematics; but any typesetting program will do. I've posted some helpful typesetting links on the class web page. I'm also happy to accept handwritten assignments, as long as they are neat and legible (see below).
- **Legibility:** If you write by hand, write your answers neatly and legibly, not too small, with as few erasures or crossouts as possible. Be sure to distinguish clearly between similar symbols, such as a/α, $b/6$, $C/\subset$, $\in/\varepsilon$, $g/q/9$, h/n, $I/l/1$, p/ρ, r/γ, $s/5$, $t/+$, $u/v/\nu$, $U/\cup$, $x/\times/\chi$, $y/4$, $z/2$, ζ/ξ, and uppercase/lowercase letters. Unless mathematical ideas spring fully and impeccably realized from your pen, your first draft is not acceptable.
- **White space:** Don't be stingy with white space. *Leave one-inch margins on all four sides of your pages.* If you don't, the grader will be annoyed because he doesn't have room to write comments, and you don't want your paper being read by an annoyed grader!

Exams:

- Midterm: to be announced.
- Final: Thursday, March 21, 8:30-10:20

During each exam, you may use two $8\frac{1}{2}'' \times 11''$ one-sided pages (or one sheet written on both sides) of your own handwritten notes. No photocopied or printed material is allowed. You may not share notes with other students.

Grading:

Before computing your final grade, I'll convert each homework score to a percentage, so that each assignment carries equal weight. Then I'll average your homework scores together, with the lowest score counting only 50% as heavily as the others.

Your grade will be based on a weighted average of the following scores:

- homework (30%)
- midterm (30%)
- final exam (40%)