Math 442 **Differential Geometry** **Winter 2013**

Assignment #1
Supplementary Exercises
(CORRECTED 1/10/2013)

S1. Suppose $f\colon \mathbb{R}^m \to \mathbb{R}^n$ is a function. We say f is an ***affine map*** if it can be written in the form $f(x) = f_0(x) + b$ for some linear map f_0 and some $b \in \mathbb{R}^n$. If f is an affine map, show that f is bijective if and only if f_0 is bijective. If this is the case, write a formula for f^{-1}.

S2. Suppose $f\colon \mathbb{R}^m \to \mathbb{R}^n$ is an affine map, written as $f(x) = f_0(x) + b$ for some linear map f_0 and some $b \in \mathbb{R}^n$. Show that for each $p \in \mathbb{R}^m$, df_p is equal to the map f_0. In particular, if f is a linear map, show that it is its own differential at each point: for all $p \in \mathbb{R}^m$, $df_p = f$.

S3. Let C be the trace of the parametrized curve σ of Example 1.1.15. Find a polynomial function $f\colon \mathbb{R}^2 \to \mathbb{R}$ such that $C = f^{-1}(\{0\})$, and determine all points $(x, y) \in C$ where $df_{(x,y)} = 0$.