Assignment #6 (CORRECTED): Due Friday, 2/24/12

Reading:

- Sections 2.5 and 2.6.

Written Assignment:

A. Exercise 2.37 (p. 42).

B. Exercise 2.43 (p. 44).

C. Suppose $C \subseteq \mathbb{R}^2$ is a simple curve. The ***generalized cylinder*** determined by C is the set $S = C \times \mathbb{R} \subseteq \mathbb{R}^3$, that is,

$$S = \{(x, y, z) \in \mathbb{R}^3 : (x, y) \in C\}.$$

Show that every generalized cylinder is a regular surface, and determine a basis for the tangent plane T_pS at an arbitrary point $p = (x_0, y_0, z_0) \in S$.

D. Let $S_2 \subseteq \mathbb{R}^3$ be the following cone:

$$S_2 = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = z^2,\ z > 0\}.$$

Show that S_2 is a regular surface, and determine a basis for the tangent plane T_pS_2 at an arbitrary point $p = (x_0, y_0, z_0) \in S_2$.

E. Let $U \subseteq \mathbb{R}^2$ be an open set, and let $S_4 = U \times \{0\} = \{(x, y, z) \in \mathbb{R}^3 : (x, y) \in U, z = 0\}$. Let $f\colon U \to \mathbb{R}$ be a smooth function, and let S_5 be the graph of f:

$$S_5 = \{(x, y, z) \in \mathbb{R}^3 : (x, y) \in U,\ z = f(x, y)\}.$$

Prove that S_4 and S_5 are diffeomorphic to each other. Determine a basis for the tangent plane T_pS_5 at an arbitrary point $p = (x_0, y_0, z_0) \in S_5$.

F. Prove that the following pairs of surfaces are diffeomorphic.

(a) The sphere $S^2 = \{(x, y, z) : x^2 + y^2 + z^2 = 1\}$ and the ellipsoid $E = \{(x, y, z) : x^2/a^2 + y^2/b^2 + z^2/c^2 = 1\}$, where a, b, c are positive constants.

(b) The plane $P = \{(x, y, z) : z = 0\}$ and the paraboloid $Q = \{(x, y, z) : z = x^2 + y^2\}$.

(c) The cylinder $Z = \{(x, y, z) : x^2 + y^2 = 1\}$ and the one-sheeted hyperboloid $H = \{(x, y, z) : x^2 + y^2 = z^2 + 1\}$.