Math 442 | **Differential Geometry** | **Winter 2011**

Assignment #3: Due 1/28/11

Reading:

- After you've read section 2.2 carefully, start skimming section 3.1. We'll skip 2.3.

Written Assignment:

A. Bär, Exercise 2.12 (page 56).

B. Bär, Exercise 2.15 (page 57). [Hint: see problem D on Assignment 1.]

C. Suppose $c, \widetilde{c} \colon I \to \mathbb{R}^2$ are two unit-speed parametrized plane curves and $\kappa, \widetilde{\kappa} \colon I \to \mathbb{R}$ are their curvatures. If $\kappa(t) = \widetilde{\kappa}(t)$ for all $t \in I$, prove that there is a Euclidean motion $F \colon \mathbb{R}^2 \to \mathbb{R}^2$ such that $F \circ c = \widetilde{c}$. [Hint: first assume that $c(t_0) = \widetilde{c}(t_0)$ and $\dot{c}(t_0) = \dot{\widetilde{c}}(t_0)$ for some $t_0 \in I$, and compute the derivative of the function $f(t) = \|\dot{c}(t) - \dot{\widetilde{c}}(t)\|^2 + \|n(t) - \widetilde{n}(t)\|^2$. To handle the general case, choose $t_0 \in I$ and show that there is a rigid motion F such that $F(c(t_0)) = \widetilde{c}(t_0)$ and $F(\dot{c}(t_0)) = \dot{\widetilde{c}}(t_0)$. The Frenet formulas of Proposition 2.2.4 might be helpful.]

D. Suppose $c \colon \mathbb{R} \to \mathbb{R}^2$ is a closed plane curve whose curvature satisfies $0 \le \kappa(t) \le 1/R$ everywhere. (Thus c is no more curved than a circle of radius R.)

   (a) If c is a simple closed curve, prove that its length is at least $2\pi R$.

   (b) If c is not simple, prove that its length is at least $2\pi R n_c$.