Math 442 **Differential Geometry** **Winter 2003**

SYLLABUS

Lectures:	MWF 12:30–1:20 Johnson 437
Instructor:	John M. Lee Padelford C-546, 543-1735 *lee@math.washington.edu* Office Hours: Mon & Fri 10:30–11:30 or by appointment
Course Web site:	*http://www.math.washington.edu/~lee/Courses/442-2003* (or from the Math Department home page, **Selected Course Web Pages → Math 442A**)

Required Textbook:

[dC] Manfredo P. do Carmo, *Differential Geometry of Curves and Surfaces*, Prentice-Hall, 1976.

Supplementary Books:

(These are all—or soon will be—on reserve in the Math Research Library, Padelford C-306. The first three are also on reserve in Odegaard.)

[S] J. Stewart, *Calculus: Early Transcendentals*, Brooks/Cole, 1999

[JRA] L. W. Johnson, R. D. Riess, and J. T. Arnold, *Introduction to Linear Algebra, 5th ed.*, Addison-Wesley, 2002

[TM] A. E. Taylor and W. R. Mann, *Advanced Calculus, 3rd ed.*, Wiley, 1983

[G] A. Gray, *Modern differential geometry of curves and surfaces*, CRC Press, 1993

[K] W. Kühnel, *Differential geometry: curves–surfaces–manifolds*, American Math. Soc., 2002

[MP] R. S. Millman and G. D. Parker, *Elements of differential geometry*, Prentice-Hall, 1977.

General description:

Broadly speaking, "differential geometry" is the use of tools of calculus to study problems in geometry. More specifically, in this course, we will apply tools of multivariable and vector calculus to study geometric properties of curves and surfaces in 3-space.

Whereas topology, the subject matter of Math 441, is concerned with properties that are preserved by homeomorphisms (continuous 1-1 correspondences with continuous inverses), geometry is concerned with properties that are preserved by distance-preserving homeomorphisms. Such properties

include distances (of course) and angles, as well as other familiar non-topological properties such as lengths of curves, areas, and volumes, all of which require calculus to compute in general. And the most important geometric property of all is one that cannot even be defined without calculus—curvature. In this course, you will learn how to use tools of multivariable calculus for studying the geometric properties of curves and surfaces.

Specifically, in 442 we will cover most or all of the following topics: the theory of curves in 3-space, the local theory of regular surfaces in 3-space, the Gauss map, curvature, geodesics, and the Gauss-Bonnet theorem. (We probabably won't get through this entire list during the winter quarter, in which case some of the topics will spill over into Math 443.)

Because of the focus on calculus, the flavor of the course will be very different from that of 441. It will be in some ways less abstract than 441, but because we will introduce quite a few new technical tools, you will need to learn at least as many new definitions, concepts, and techniques as in 441. The payoff is that you should end up having mastered the tools needed to delve into one of the most beautiful and physically relevant branches of modern mathematics.

Prerequisites:

The official prerequisite is grades of 2.0 or higher in Math 326, 328, and 308. (Note that 318 may be substituted for 308, or 335 may be substituted for all three prerequisites.) However, grades of 3.0 or higher in all of these courses would be more realistic. In addition, the most important prerequisite is a genuine interest in abstract mathematics, or at least a mind open to the possibility. Intellectual curiosity and an ability to express mathematics clearly in writing are also important. Of course, one purpose of the course is to improve such skills.

Specific prerequisite material includes the following. If two or three items on the list are unfamiliar, you are probably still OK as long as you're willing to spend some extra time reviewing. If there are half a dozen unfamiliar items on the list, you're not ready for this class. Of course many people will be rusty on various topics. Some of these topics will be reviewed briefly in class as they arise, but be prepared to do some review on your own.

- Vector calculus: partial derivatives, the chain rule, dot products, cross products, tangent lines/planes, line integrals, surface integrals, gradients, vector fields. [S, Chapters 14, 15, 16.]
- Analysis in several variables: differentials, Jacobians, the inverse and implicit function theorems. [TM, Chapters 6, 7, 8, 9.]
- Point sets and continuous functions in the real line and the plane: open and closed sets, boundaries, limit points, closures, continuous maps, connected sets, compact sets. [TM, Chapters 5, 16, 17.]
- Linear algebra: vector spaces, bases and dimension, linear transformations and their representation by matrices, determinants, matrix algebra, eigenvectors and eigenvalues, inner products. [JRA, Chapters 1, 2, 3, 4.]

Note that Math 441 (Topology) is not prerequisite for Math 442. However, if you haven't had 441, you should be quite sure that your understanding of the material described above on point sets and continuous functions is solid.

Homework:

A homework assignment will be given out each week. Usually, each assignment will be handed out on a Monday and will be due the following Monday. When Monday is a UW holiday, the assignment will be handed out or due the following Wednesday. Graded assignments will usually be returned at the next lecture, along with written solutions to selected problems.

Plan to spend a lot of time on homework—six hours or more outside of class during most weeks. A typical homework assignment will consist of the following:

I. **Reading:** Typically, you will be given approximately two sections of the textbook to read each week. This will usually correspond to the material that is being discussed in lecture that week. I will expect you to read through the chapter quickly before the relevant lectures, and then to reread it carefully after the lectures.

II. **Practice Problems:** Most assignments will include a number of "practice problems." These are not to be handed in for a grade, but I expect you to do (or at least figure out how to do) all of them for your own good. Understanding these problems will be important for solidifying your understanding of the text and lectures, and for preparing to do the required problems. Some of these problems are likely to show up on exams.

III. **Required Problems:** The problems listed as "Required Problems" are for you to write up and hand in for a grade. These problems, consisting almost entirely of proofs, are the heart of the course, and they will constitute a significant part of your course grade.

Notice that the first homework assignment consists mainly of material that is prerequisite to this course, and which you should already have mastered in previous courses. Most of you will have to brush up on some of these concepts before you're able to do the homework; but if you find that you really can't handle the first assignment at all, then you might not have sufficient preparation for this course.

I encourage you to form study groups and work together on the homework problems (it's usually the best and fastest way to learn). However, when you write up your solutions to hand in, *you must write your own solutions in your own words.*

Here are some other important instructions regarding homework:

- Please staple your homework papers together, with the required problems in numerical order, and with your name and student number on every page. Write legibly, in complete sentences, and leave ample margins in which the grader can write comments.
- Each of the "Required Problems" will be graded. Selected problems will be graded on a scale of 0 to 10 points, and the remaining ones will be graded on a scale of 0 to 2. The 10-point

problems will be graded in detail, with partial credit given for incomplete solutions. The 2-point problems will be given 0 points if there is no reasonable attempt at a solution, 1 point if you make a reasonable attempt, and 2 points if there are no blatant mistakes. Since you won't know in advance which problems will be graded for 10 points, it pays to try to answer all problems as thoroughly as possible.

- In computing your final grade, your raw homework scores will be converted to percentages, so that each assignment carries equal weight. Then your lowest homework score will be dropped, and the remaining assignments will be averaged.

Exams:

- **Midterm: Monday, February 10, 12:30–1:20pm, Johnson 437**.
- **Final: Thursday, March 20, 8:30–10:20am, Johnson 437**

During each exam, you may use two $8\frac{1}{2}'' \times 11''$ one-sided pages (or one sheet written on both sides) of your own handwritten notes. No photocopied or printed material is allowed. You may not share notes with other students.

Grading:

Your grade will be based on homework (30%), the midterm (30%), and the final exam (40%). If your final exam grade is higher than your midterm grade, then your final exam grade will replace your midterm grade, so in that case your final exam will be worth 70% of your grade. Also, doing the homework is a prerequisite for passing the class, regardless of how well you do on the exams.