Math 441 **Topology** **Fall 2012**

Assignment #7 Supplement (CORRECTED)

Exercise S4: Suppose X is a compact topological space and $f\colon X \to \mathbb{R}$ is a continuous function that is everywhere positive: $f(x) > 0$ for all $x \in X$. Show that there is some positive number ε such that $f(x) \geq \varepsilon$ for all $x \in X$. Give a counterexample when X is not compact.

Exercise S5: Suppose X is a noncompact Hausdorff space. Let ∞ be some object not in X, and let $X^* = X \cup \{\infty\}$. Define a topology on X^* by declaring the open sets to be of the following two types:

(i) Open subsets of X (in its given topology);
(ii) Sets of the form $\{\infty\} \cup (X \smallsetminus K)$, where K is a compact subset of X.

(You may accept the fact that this is a topology.) Prove that X^* is compact. (The space X^* is called the ***one-point compactification of X***.)

Exercise S6: Prove that the one-point compactification of $\mathbb{R}^2$ is homeomorphic to S^2. [Hint: Define $f\colon (\mathbb{R}^2)^* \to S^2$ by

$$f(u,v) = \left(\frac{2u}{u^2+v^2+1}, \frac{2v}{u^2+v^2+1}, \frac{u^2+v^2-1}{u^2+v^2+1} \right), \qquad (u,v) \in \mathbb{R}^2;$$
$$f(\infty) = (0,0,1),$$

and use the closed map lemma to show that f is a homeomorphism.]