Math 441 **Topology** **Autumn 2002**

Assignment #7: Due 11/18/02

I. Reading:

- Read Patty, §2.2.
- Skim Patty, §2.6. *(We will not cover §§2.3 through 2.5.)*

II. Practice problems:

1. Patty, Exercises 2.1 (pp. 65–67) #1, 2, 6, 7, 16, 17, 21.

III. Required problems:

1. Patty, Exercises 2.1 (pp. 65–67) #4.
2. Patty, Exercises 2.1 (pp. 65–67) #5.
3. Patty, Exercises 2.1 (pp. 65–67) #8.
4. Patty, Exercises 2.1 (pp. 65–67) #9.
5. Patty, Exercises 2.1 (pp. 65–67) #15.
6. Patty, Exercises 2.1 (pp. 65–67) #22.
7. (a) Prove the following generalization of the pasting lemma (Theorem 2.15): *Let X and Y be topological spaces, let $A_1, \dots, A_k$ be closed subsets of X such that $X = A_1 \cup \cdots \cup A_k$, and for each $i = 1, \dots, k$, let $f_i \colon A_i \to Y$ be a continuous function such that $f_i|_{A_i \cap A_j} = f_j|_{A_i \cap A_j}$ for each i and j. Then there is a unique continuous function $h \colon X \to Y$ such that $h|_{A_i} = f_i$ for each i.*

   (b) By considering the space $X = [0,1] \subset \mathbb{R}$ with the usual topology, and the subspaces $A_0 = \{0\}$, $a_i = [1/(i+1), 1/i]$ for $i = 1, 2, \dots$, show that the previous result is false if the sets $\{A_1, \dots, A_k\}$ are replaced by an infinite sequence of closed sets.