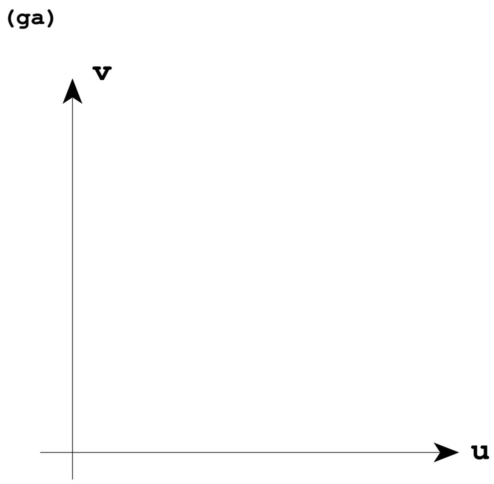
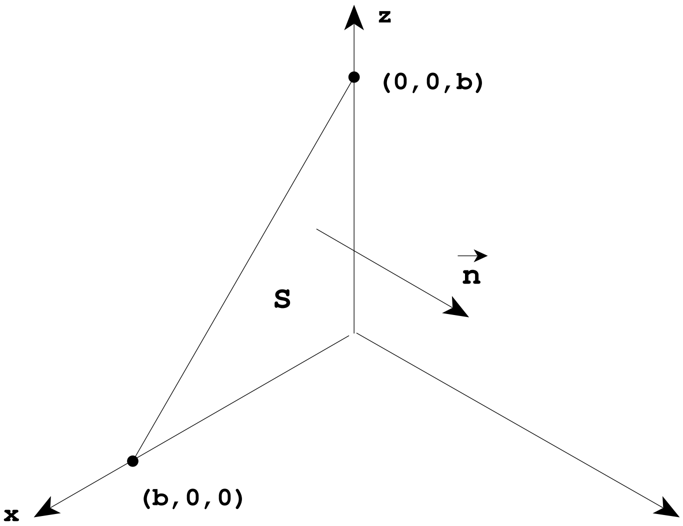
Math 324A & 324B Advanced Multivariable Calculus I Spring 1997

WORKSHEET #6: 5/14/97
REVIEW PROBLEMS FOR TEST #4

1. (*Comprehensive review problem on flux*) The surface S is the triangle in the xz-plane shown below right. We orient S by choosing our normal in the $\vec{j}$ direction.





The vector field $\overrightarrow{F}$ is given by $\overrightarrow{F} = (2x - 3y + 2z, x + 7y + 2z, 6x - y - 3z)$.

(a) Write down a parameter representation of S with the following property: The point $(b, 0, 0)$ corresponds to $(u, v) = (0, 0)$, $(0, 0, 0)$ to $(u, v) = (1, 0)$, and $(0, 0, b)$ to $(u, v) = (0, 1)$.

(b) Draw in the uv-plane above left the parameter domain $S°$ that corresponds to S.

(c) Find the standard normal of the parametrization you wrote down in (a). Does the standard normal agree with the given orientation or not?

(d) Write down the formula for the normal component F_n in terms of (u, v).

(e) The normal component F_n is a function $F_n(u, v)$. Find the maximum and minimum values $F_n(u, v)$ takes on $S°$. [Hint: You will have to find the extrema of a linear function of two variables. Look at the level curves of the function and use ruler and pencil.]

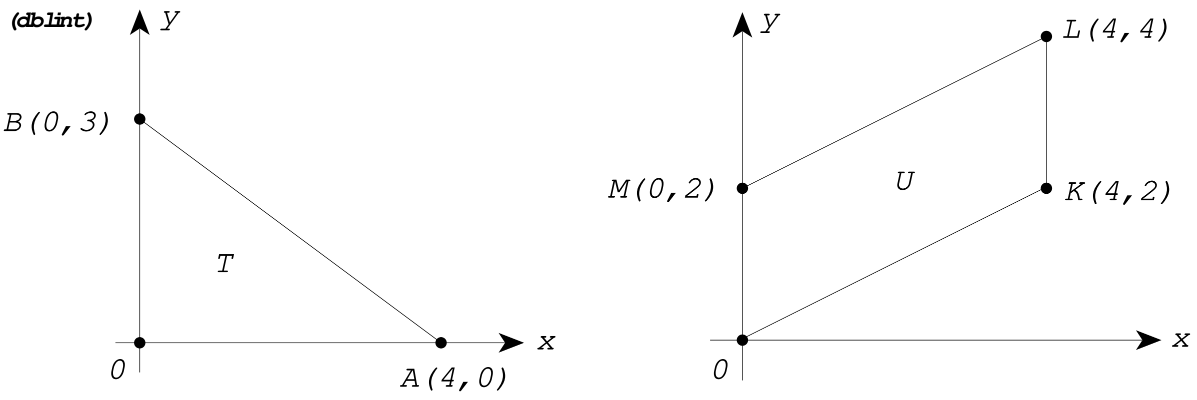
(f) Estimate $\iint_S \overrightarrow{F} \cdot d\vec{A}$. That is, find numbers m and M so that

$$m \leq \iint_S \overrightarrow{F} \cdot d\vec{A} \leq M.$$

(g) Set up $\iint_S \overrightarrow{F} \cdot d\vec{A}$ as a double integral in the uv-plane.

(h) Evaluate the integral.

2. (*Review of double integrals in the plane*) We consider a triangle T (below left) and a parallelogram U (below right). By "triangle" we mean "the points on the edges OA, AB, BO together with the points in the interior," and similarly for "parallelogram."



(a) The function $f(x,y)$ is defined by $f(x,y) = y$. Find the double integral of $f(x,y)$ over the triangle T shown above left. That is, evaluate $\iint_T f(x,y)\,dx\,dy$. Your answer has to be a number.

(b) Set up the double integral of a generic function $g(x,y)$ over the parallelogram U shown above right. [Note: You have to decide in which order to integrate. Use the usual way of indicating the order of integration: $\iint g(x,y)dx\,dy$ means "first integrate with resect to x, then y", etc. To avoid confusion, be sure to name the varaiables in the limits of integration. For example, write $\int_{y=0}^{y=3}$, not just $\int_0^3$.]