**Math 324A & 324B Advanced Multivariable Calculus I** **Spring 1997**

ANSWERS TO WORKSHEET #6

1. (a) $(x, y, z) = (b, 0, 0) + u(-b, 0, 0) + v(-b, 0, b)$

   (b) $S^0$ is the triangle in the $uv$-plane with vertices $(0,0), (1,0), (0,1)$.

   (c) The standard normal has components $(0, b^2, 0)$ and points in the same direction as the given normal $\vec{n} = \vec{j}$.

   (d) $F_n = b(1 - u + v)$

   (e) Minimum 0 at $(u, v) = (1, 0)$, maximum $2b$ at $(u, v) = (0, 1)$

   (f) $A(S) = \frac{1}{2}b^2$; $m = (0)A(S) = 0$, $M = (2b)A(S) = b^3$

   (g) $\overrightarrow{F} \cdot (\vec{r}_u \times \vec{r}_v) = b^3(1 - u + v)$; $\displaystyle\int_{v=0}^{v=1} \int_{u=0}^{u=1-v} b^3(1 - u + v)\, du\, dv$.
   [It can also be done by constant-$u$ slices.]

   (h) $$b^3 \int_{v=0}^{v=1} \left( \int_{u=0}^{u=1-v} (1 - u + v)\, du \right) dv = b^3 \int_{v=0}^{v=1} \left( (u - \tfrac{1}{2}u^2 + vu)\Big|_{u=0}^{u=1-v} \right) dv$$
   $$= b^3 \int_{v=0}^{v=1} (\tfrac{1}{2} + v - \tfrac{3}{2}v^2)\, dv = b^3(\tfrac{1}{2}v + \tfrac{1}{2}v^2 - \tfrac{1}{2}v^3)\Big|_{v=0}^{v=1} = \tfrac{1}{2}b^3.$$

2. (a) This integral can be done either by constant-$x$ slices or constant-$y$ slices:

$$\int_{y=0}^{y=3} \left( \int_{x=0}^{x=(1/3)(12-4y)} y\, dx \right) dy = \int_{y=0}^{y=3} y\frac{1}{3}(12 - 4y) dy = \frac{1}{3}(6y^2 - \frac{4}{3}y^3)\Big|_{y=0}^{y=3} = 6$$

   or

$$\int_{x=0}^{x=4} \left( \int_{y=0}^{y=(1/4)(12-3x)} y\, dy \right) dx = \int_{x=0}^{x=4} \left(\frac{1}{4}\right)^2 \frac{1}{2}(12 - 3x)^2\, dx$$
$$= \frac{1}{32}\left(144x - 36x^2 + 9\frac{x^3}{3}\right)\Big|_{x=0}^{x=4} = 6.$$

   (b) The limits of integration are much easier to figure out if you use constant $x$-slices:

$$\int_{x=0}^{x=4} \int_{y=(1/2)x}^{y=(1/2)x+2} g(x, y)\, dy\, dx.$$