Math 324A & 324B **Advanced Multivariable Calculus I** **Spring 1997**

WORKSHEET #5: 5/1/97

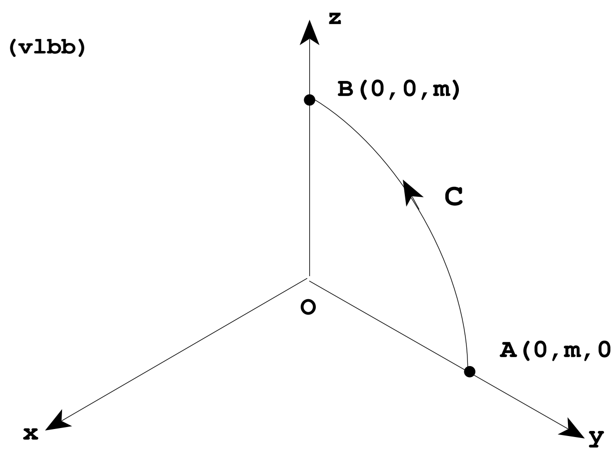
REVIEW PROBLEMS FOR TEST #3

1. $\overrightarrow{F}$ is the constant vector field $\overrightarrow{F} = (1, 4, 2)$, and C is the curve $(x, y, z) = (4t^2, 3t^3, t+1)$ from $A(4, 3, 2)$ to $B(16, 24, 3)$. Evaluate the line integral.

2. Can you find a potential for the following four vector fields? If Yes, find a potential. If No, explain.

$$\overrightarrow{F} = (xy^2, x^2y^2) \qquad \overrightarrow{H} = \text{grad}(2x^2 - y^2 + 3xy)$$
$$\overrightarrow{K} = (xy^2, x^2y) \qquad \overrightarrow{L} = \text{grad}\,\frac{1}{x-y+2}$$

3. In each of the questions below, C is one fourth of the circle of radius m in the yz-plane, with center at the origin and with the orientation shown.



(a) The vector field $\overrightarrow{F}$ is given by $\overrightarrow{F} = (8x + 3z - y, 2y - x + 7z, 6z + 3x + 7y)$. Evaluate $\int_C \overrightarrow{F} \cdot d\vec{r}$.

(b) The vector field $\overrightarrow{G}$ is given by $\overrightarrow{G} = \text{grad}(e^{x+2y+3z})$. Evaluate $\int_C \overrightarrow{G} \cdot d\vec{r}$.

(c) The vector field $\overrightarrow{H}$ is given by $\overrightarrow{H} = \frac{(x,y,z)}{x^2+y^2+z^2}$. The curl of $\overrightarrow{H}$ is the zero vector. Evaluate $\int_C \overrightarrow{H} \cdot d\vec{r}$. [Hint: Do not try to find a potential.]

Topics for Test #3

Line integrals

- Evaluating by parametrizing the curve (Section 7)
- Evaluating and estimating by using the tangential component (Section 7)
- Evaluating using a potential if there is one (Section 8)
- Evaluating along a simpler curve using path independence (Section 9)

Conservative vector fields

- Path independence and integrals around closed curves (Section 8)
- The curl test (Section 9)
- Finding a potential (Section 9)