Math 324A & 324B Advanced Multivariable Calculus I **Spring 1997**

ANSWERS TO WORKSHEET #5

1. The function $f(x,y,z) = x + 4y + 2z$ is a potential for $\overrightarrow{F}$. The line integral equals $f(16,24,3) - f(4,3,2) = 98$.

2. The curl of $\overrightarrow{F}$ is different from zero. That is, $\overrightarrow{F}$ is not conservative and does not have a potential. For $\overrightarrow{H}$, a potential is already given: $h(x,y) = 2x^2 - y^2 + 3xy$, and similarly $l(x,y) = 1/(x - y + 2)$ is a potential for $\overrightarrow{L}$. The field $\overrightarrow{K}$ has a potential $k(x,y) = (1/2)x^2y^2$.

3. (a) The field $\overrightarrow{F}$ is conservative. The straightforward approach is to use the "compute-and-check" method to find that $f(x,y,z) = 4x^2 + y^2 + 3z^2 + 3xz - xy + 7yz$ is a potential for $\overrightarrow{F}$. Therefore the line integral is equal to $f(0,0,m) - f(0,m,0) = 3m^2 - m^2 = 2m^2$. Another approach that is a bit easier is to replace C with an easier path from A to B: from A to O along the y-axis and then from O to B along the z-axis. Along AO the line integral reduces to $\int_m^0 2y\,dy = -m^2$, and along OB we get $\int_0^m 6z\,dz = 3m^2$, and the sum is $2m^2$.

   (b) $f(x,y,z) = e^{x+2y+3y}$ is a potential, and $f(0,0,m) - f(0,m,0) = e^{3m} - e^{2m}$.

   (c) The curl test for vector fields in space tells you that $\overrightarrow{F}$ is conservative. What about an easier path? We cannot take the easier path we took in (a) because the vector field is not defined at the origin. What about F_{tan}? It is zero because along C the field vector is perpendicular to C. The line integral is zero.