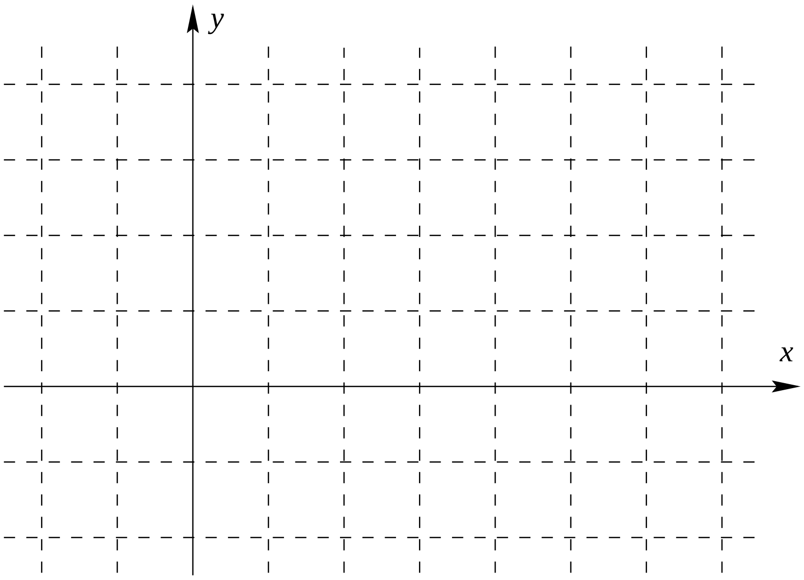
Math 324A & 324B **Advanced Multivariable Calculus I** **Spring 1997**

WORKSHEET #4: 4/16/97
REVIEW PROBLEMS FOR TEST #2

1. The function $h(x, y)$ is defined by $h(x, y) = 4x - 2y + 3$. We consider the vector field $\overrightarrow{F} = \operatorname{grad} h$.

   (a) K is the level curve of $h(x, y)$ of level -3. Draw K in the system below and label it clearly.

   (b) In the system below draw the field vector of $\overrightarrow{F}$ at $P(-1, 1)$. Label P and $\overrightarrow{F}(p)$ clearly.



   (c) We consider $\overrightarrow{F}$ as a velocity field of a steady state flow. At time $t = 0$ we drop a particle into the flow at $Q(1, 3)$. At time $t = 1.5$ the particle is at the point R. Draw the point R and label it clearly.

   (d) At time $t = 2$, another particle is at the point $S(6, -1)$. Find the functions $(a(t), b(t))$ that give the location of the particle at time t. [Hint: Find the location of the particle at time $t = 0$.]

   (e) Compute the curl and the divergence of $\overrightarrow{F}$ at the points $P(-1, 1)$, $Q(1, 3)$, and $S(6, -1)$.

2. $\overrightarrow{G}$ is a vector field in space. We do not know the formulas for the components (G_1, G_2, G_3), but we are told that the following curve C is one of the flow lines:

$$C : (x, y, z) = (t^2, 4t + 5, t^3 + 4).$$

At time $t = 1$ sec, the point on C has coordinates $(1, 9, 5)$. Can you find the field vector at the point $(1, 9, 5)$? If Yes, find it. If No, use one or more complete sentences to explain.

3. We consider a vector field $\overrightarrow{F}$ in the plane. It is defined by $\overrightarrow{F} = (x + y, -3x - y)$.

   (a) Find the components of the field vector at $(x, y) = (3, 2)$.

   (b) The curve C: $(x, y) = (a(t), b(t))$ is a flow line of $\overrightarrow{F}$. It describes the movement of a particle which at time $t = 3$ seconds passes through the point $Q(4, 6)$.

   Do we have enough information to find $(a'(3), b'(3))$ without solving the system of differential equations? If Yes, find $(a'(3), b'(3))$. If No, use one or more complete sentences to explain.

   (c) K is a different curve. It is given by $(x, y) = (p(t), q(t))$ where $p(t)$ and $q(t)$ are defined as follows:

$$\begin{aligned} p(t) &= \cos(\sqrt{2}\,t) + \sin(\sqrt{2}\,t) \\ q(t) &= (\sqrt{2} - 1)\cos(\sqrt{2}\,t) - (\sqrt{2} + 1)\sin(\sqrt{2}\,t). \end{aligned}$$

   Can you decide whether K is a flow line of our vector field $\overrightarrow{F}$? No matter what your answer is, use one or more complete sentences to justify your answer.

   (d) Compute the curl and divergence of $\overrightarrow{F}$ at an arbitrary point (x, y).

4. The function $h(x, y, z)$ is defined by $h(x, y, z) = xyz + 2x + 3y + 4z$, and $\overrightarrow{G}$ is the vector field in space defined by $\overrightarrow{G} = \operatorname{grad} h$.

   (a) Write down the system of ODE you'd have to solve to find the equations of flow lines. You don't have to solve the system.

   (b) Compute $\nabla\overrightarrow{G}$, $\nabla \cdot \overrightarrow{G}$, and $\nabla \times \overrightarrow{G}$.

5. The vector field $\overrightarrow{F}$ is given by $\overrightarrow{F} = (2x, 0, x)$.

   (a) Write down the system of ODE you have to solve to find flow lines.

   (b) Find the equation of the flow line that passes through the point $(5, 5, 5)$ at time $t = 2$.

   (c) Compute $\nabla \cdot \overrightarrow{F}$ and $\nabla \times \overrightarrow{F}$ at an arbitrary point (x, y, z).