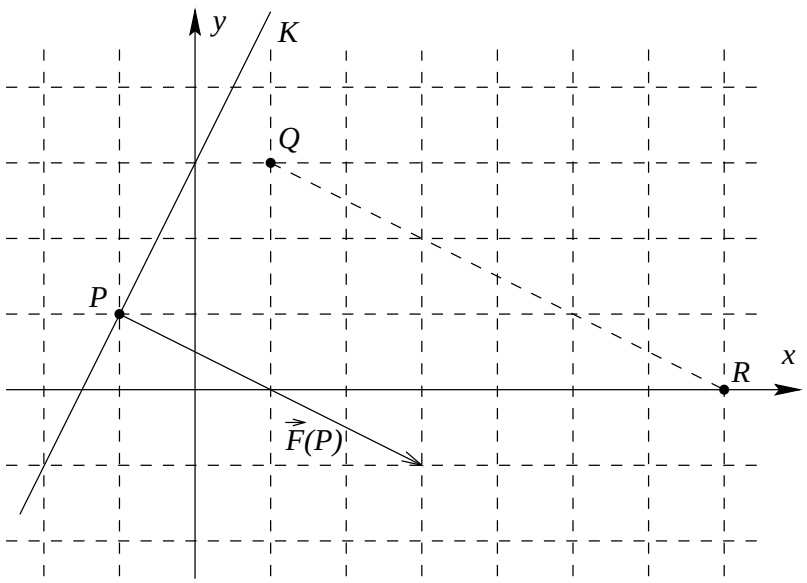
Math 324A & 324B Advanced Multivariable Calculus I **Spring 1997**

ANSWERS TO WORKSHEET #4

1. (a)–(c): See below.



(d) The velocity vector is the constant vector $(4,-2)$. At $t=2$ the particle is at $(6,-1)$. To find the location at $t=0$ we have to go back from $(6,-1)$ twice the velocity vector. Thus, at $t=0$ the particle is at the point $(6,-1)-2(4,-2)=(-2,3)$. At time t the particle is at the point $(x,y)=(-2,3)+t(4,-2)$. (e) curl $\vec{F}=(0,0,0)$; div $\vec{F}=0$.

2. The velocity vector of the particle at time $t=0$ is $(2t,4,3t^2)|_{t=1}=(2,4,3)$, and this is the field vector at $(1,9,5)$.

3. (a) $(5,-11)$. (b) The velocity vector $(a'(3),b'(3))$ at the point $(4,6)$ must be the field vector at $(4,6)$, and at this point the components of the field vector are $(10,-18)$. (c) To find out whether K is a flow line we have to check whether the functions $x(t)=p(t)$ and $y(t)=q(t)$ are solutions of the following system of ODE:

$$x'(t) = x(t)+y(t)$$
$$y'(t) = -3x(t)-y(t).$$

If you work out the expressions on the left and the right side, you see that the function $p(t)$ and $q(t)$ are solutions of the system. That is, K must be a flow line of $\vec{F}$. (d) curl $\vec{F}=-4$ (or $-4\vec{k}$); div $\vec{F}=0$.

4. (a) The component functions of the vector field are $(yz+2, xz+3, xy+4)$. To find flow lines we would have to solve the system

$$\begin{aligned} x'(t) &= y(t)\,z(t) + 2 \\ y'(t) &= x(t)\,z(t) + 3 \\ z'(t) &= x(t)\,y(t) + 4. \end{aligned}$$

(b) $\nabla \overrightarrow{G}$ doesn't make sense; $\nabla \cdot \overrightarrow{G} = 0$; and $\nabla \times \overrightarrow{G} = (0,0,0)$.

5. (a) The system is

$$\begin{aligned} x'(t) &= 2x(t) \\ y'(t) &= 0 \\ z'(t) &= x(t). \end{aligned}$$

(b) $x(t) = 5e^{-4}e^{2t}$, $y(t) = 5$, $z(t) = (2.5)e^{-4}e^{2t} + 2.5$. (c) $\nabla \cdot \overrightarrow{F} = 2$; $\nabla \times \overrightarrow{F} = (0,-1,0)$.