Name: ______________________________ **Student Number:** ______________

Math 324B **Advanced Multivariable Calculus I** **Spring 1997**

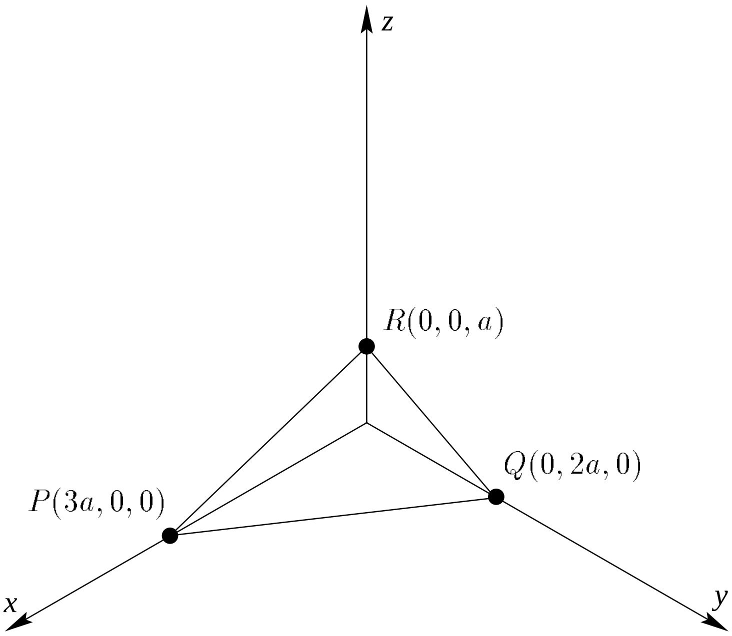
Test #4: 5/16/97

- There are 5 problems at 20 points each, for a total of 100 points.
- You may use two $8\frac{1}{2}'' \times 11''$ one-sided pages (or one sheet written on both sides) of your own handwritten notes (no photocopied or printed material).
- *Show your work:* If your work demonstrates that you had the right idea, you might get partial credit even if your answer is wrong. On the other hand, if your answer is not justified by your work, you might lose points even if your answer is correct. If you need more room, continue your work on the back of the page.
- Use the boxes for answers when possible. Measurements must be made with a ruler, and lines and vectors must be drawn with a ruler.
- Simplify algebraic and numerical expressions in answers to the point where they can be evaluated by a student in a precalculus course with a calculator, but you do not need to take them any further. Leave answers such as $\sqrt{7+(6.8)^2}$ as they are, but work out vector expressions such as dot products and cross products. Calculators are allowed, but you won't need one; it is preferable to leave numerical answers in unsimplified form. Laptop computers are not allowed.
- If you have a question, raise your hand and I will come to your seat.

For grader's use only

1:	
2:	
3:	
4:	
5:	
Total:	

1. Suppose S is the triangle with vertices, $P(3a, 0, 0)$, $Q(0, 2a, 0)$, and $R(0, 0, a)$ shown below.



(a) Write down a parameter representation of S such that P corresponds to $(u, v) = (1, 0)$, Q corresponds to $(u, v) = (0, 0)$, and R corresponds to $(u, v) = (0, 1)$.

$x =$ ☐

$y =$ ☐

$z =$ ☐

(b) We orient the triangle S by the normal that points toward the origin. Suppose $\overrightarrow{F}$ is the constant vector field $\overrightarrow{F} = (2, 3, 0)$. Find the flux of $\overrightarrow{F}$ across S.

flux = ☐

2. The surface S is given by $(x, y, z) = (uv, u + v, v^2)$. The point Q on S corresponds to the parameters $(u, v) = (2, 1)$.

(a) Through Q passes a coordinate curve $v = \mathit{constant}$. Write down a parameter representation $(x, y, z) = (\ \dots\ ,\ \dots\ ,\ \dots\ )$ of this coordinate curve.

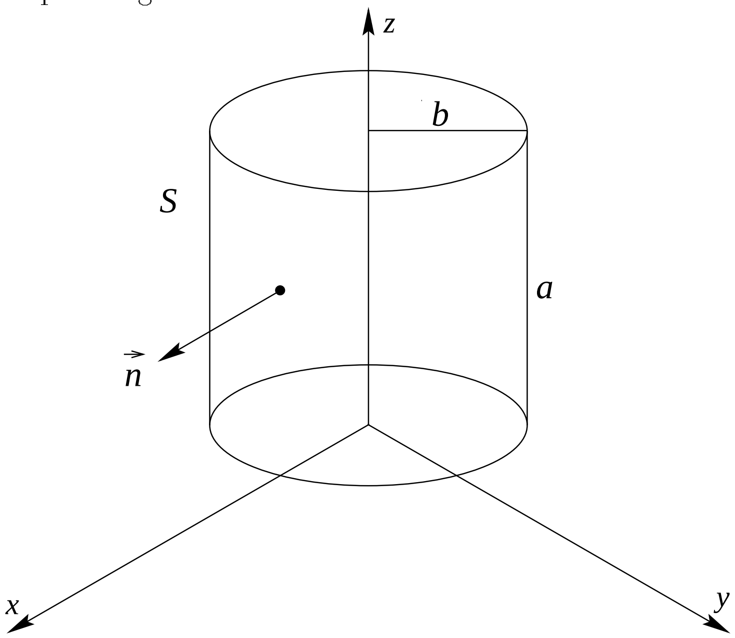
$(x, y, z) =$ $\boxed{}$

(b) Write down a parameter representation of the tangent plane to S at the point Q. Use p, q for the parameters and write your answer in the form

$$(x, y, z) = (\dots) + p(\dots) + q(\dots).$$

$(x, y, z) =$ $\boxed{}$

3. S is the cylindrical surface of radius b and height a, centered around the z axis, as shown below. (S contains only the cylinder, not the top and bottom caps.) We orient S by the outward-pointing normal.



The vector field $\overrightarrow{F}$ is given by $\overrightarrow{F} = (xz^2, yz^2, 0)$.

(a) Find a formula for F_n, the normal component of the vector field $\overrightarrow{F}$.

$F_n =$ $\boxed{}$

(b) Give an estimate of the flux of $\overrightarrow{F}$ across S. That is, find constants c and C so that

$$c \le \iint_S \overrightarrow{F} \cdot d\overrightarrow{A} \le C.$$

Hint: The surface area of a cylinder of radius r and height h is $2\pi rh$.

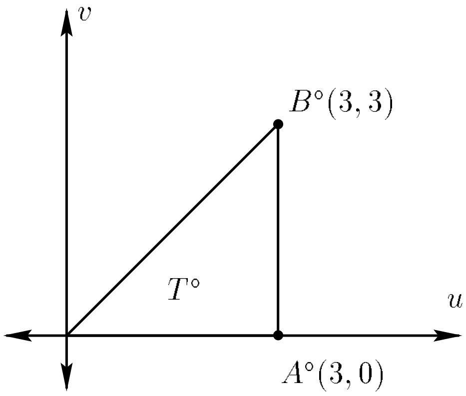
$c =$ $\boxed{}$

$C =$ $\boxed{}$

4. Suppose that α is the plane given by $(x, y, z) = (u, v, 3u - v)$. We take the normal whose z-component is positive. Suppose that $\overrightarrow{F}$ is the vector field given by

$$\overrightarrow{F} = (4y, 2x, z - 2x).$$

Let T° be the triangle $OA^\circ B^\circ$ in the uv-plane as shown below. Let T be the corresponding piece of the plane α. Set up the integral to compute the flux of $\overrightarrow{F}$ across T.

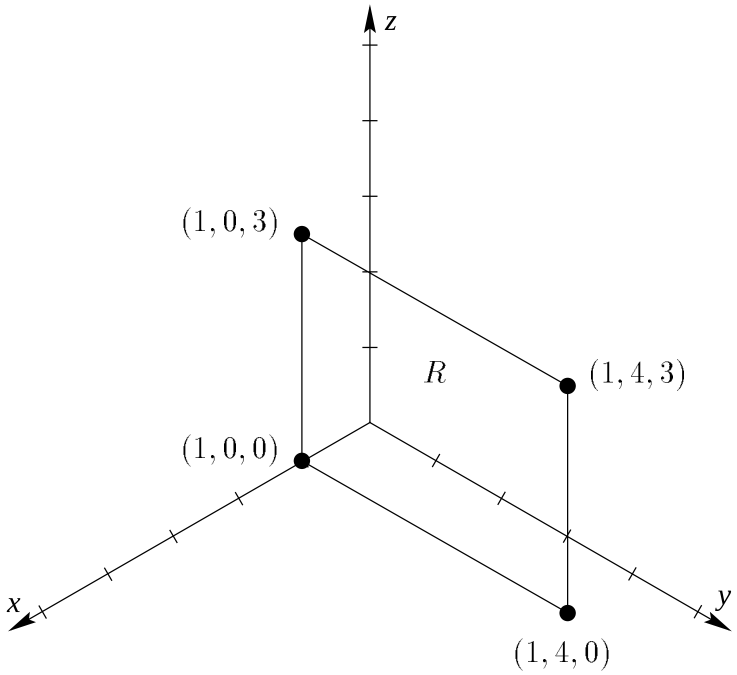


In the boxes for the limits, you must name the variables. For example, if one of the u-limits is 6*, you must write* $u = 6$*, and writing only* 6 *is not enough.*

$$\int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \boxed{} \, d\boxed{} \, d\boxed{}$$

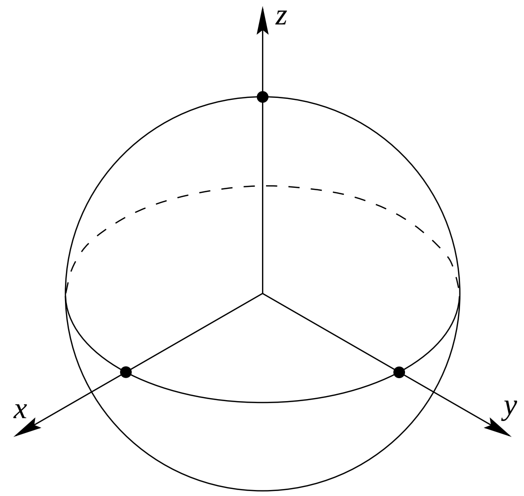
5. Compute the flux of each vector field across the given surface. *Hint: Look at the normal components of the vector fields. The surface area of a sphere of radius b is $4\pi b^2$.*

(a) $\overrightarrow{F}$ is the vector field $\overrightarrow{F} = (5, 3y^2, 3xyz)$, and R is the rectangle in the plane $x = 1$ shown below. We take the normal to R to be $-\vec{\imath}$.



Flux of $\overrightarrow{F}$ across R: $\boxed{}$

(b) $\overrightarrow{G}$ is the vector field $\overrightarrow{G} = (3y, -3x, 0)$, and S is the sphere of radius b centered at the origin, as shown below. Orient S with the outward-pointing normal.



Flux of $\overrightarrow{G}$ across S: $\boxed{}$