Name: ________________________________ **Student Number:** ______________

Math 324A **Advanced Multivariable Calculus I** **Spring 1997**

Test #3: 5/2/97

- There are 5 problems at 20 points each, for a total of 100 points.
- You may use two $8\frac{1}{2}'' \times 11''$ one-sided pages (or one sheet written on both sides) of your own handwritten notes (no photocopied or printed material).
- *Show your work:* If your work demonstrates that you had the right idea, you might get partial credit even if your answer is wrong. On the other hand, if your answer is not justified by your work, you might lose points even if your answer is correct. If you need more room, continue your work on the back of the page.
- Use the boxes for answers when possible. Measurements must be made with a ruler, and lines and vectors must be drawn with a ruler.
- Simplify algebraic and numerical expressions in answers to the point where they can be evaluated by a student in a precalculus course with a calculator, but you do not need to take them any further. Leave answers such as $\sqrt{7+(6.8)^2}$ as they are, but work out vector expressions such as dot products and cross products. Calculators are allowed, but you won't need one; it is preferable to leave numerical answers in unsimplified form. Laptop computers are not allowed.
- If you have a question, raise your hand and I will come to your seat.

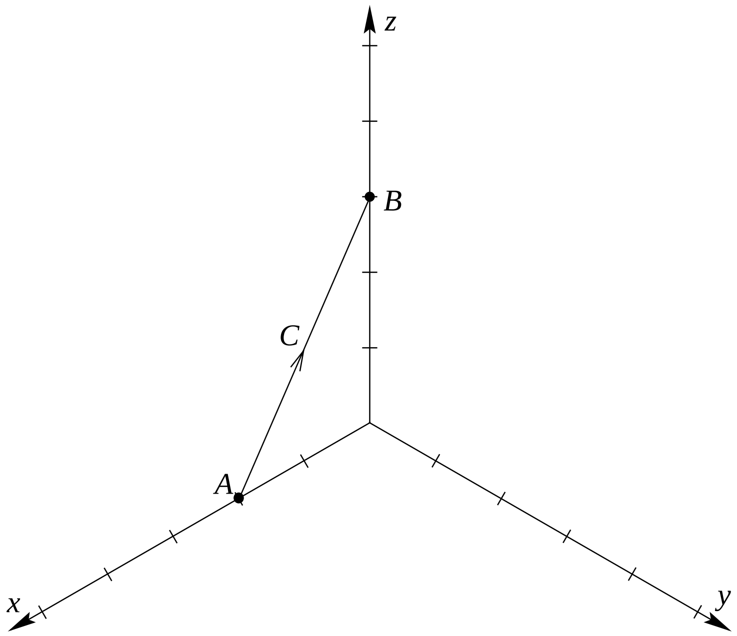
For grader's use only

1:	
2:	
3:	
4:	
5:	
Total:	

1. Let $\overrightarrow{F}$ be the vector field in space given by

$$\overrightarrow{F} = (x, z^2, -y^2).$$

Let C be the line segment from $A(2,0,0)$ to $B(0,0,3)$.



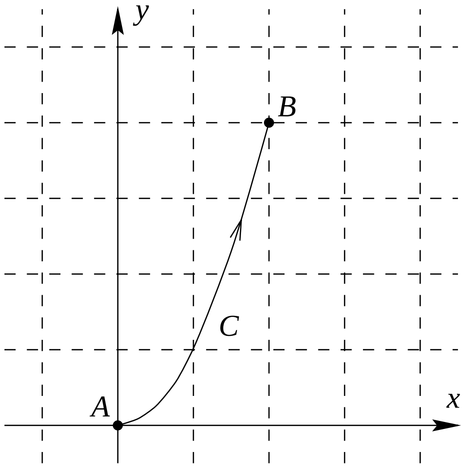
Evaluate $\int_C \overrightarrow{F} \cdot d\vec{r}$.

$$\int_C \overrightarrow{F} \cdot d\vec{r} = \boxed{}$$

2. Let $\overrightarrow{F}$ be the vector field in the plane given by

$$\overrightarrow{F} = (y^2, 2xy).$$

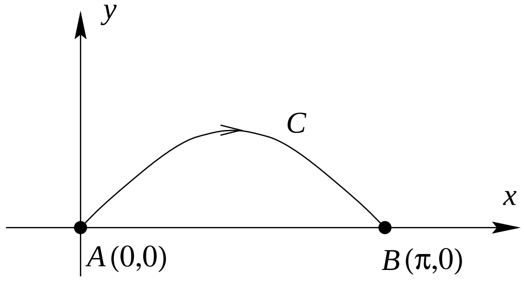
Let C be the portion of the curve $(x, y) = (t, t^2)$ from $(0, 0)$ to $(2, 4)$ with the orientation shown below.



Compute $\int_C \overrightarrow{F} \cdot d\vec{r}$.

$$\int_C \overrightarrow{F} \cdot d\vec{r} = \boxed{}$$

3. In each of the two parts of this problem, C is the portion of the plane curve $y = \sin x$ from $x = 0$ to $x = \pi$ (see below).



(a) $\overrightarrow{G}$ is the vector field

$$\overrightarrow{G} = \operatorname{grad} e^{2x+y}.$$

Evaluate $\int_C \overrightarrow{G} \cdot d\vec{r}$.

$$\int_C \overrightarrow{G} \cdot d\vec{r} = \boxed{}$$

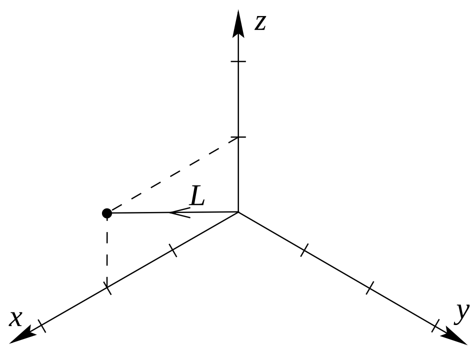
(b) $\overrightarrow{F}$ is the vector field

$$\overrightarrow{F} = \left(\frac{y}{1 + x^2y^2}, \frac{x}{1 + x^2y^2} \right).$$

Its curl is zero. Evaluate $\int_C \overrightarrow{F} \cdot d\vec{r}$.

$$\int_C \overrightarrow{F} \cdot d\vec{r} = \boxed{}$$

4. Let L be the line segment $(x, y, z) = (2t, 0, t)$ from the origin to $P(2, 0, 1)$, and let $\overrightarrow{F} = (e^{x^2}, e^{2y^2}, e^{4z^2})$.



(a) Compute the tangential component F_{tan} of $\overrightarrow{F}$ along C. Your answer should be a function of t.

$F_{tan} =$ $\boxed{}$

(b) Using your answer to (a), estimate the line integral $\int_C \overrightarrow{F} \cdot d\vec{r}$. This means find constants k and K such that

$$k \leq \int_C \overrightarrow{F} \cdot d\vec{r} \leq K.$$

$k =$ $\boxed{}$

$K =$ $\boxed{}$

5. Apply the curl test to each of the following vector fields to determine whether it is conservative or not. In each case, circle YES if it is definitely conservative, NO if it is definitely not conservative, or ??? if it is impossible to decide from the curl test. For those that are conservative, compute a potential.

(a) $\overrightarrow{G} = (x^3 - 2xy, y^2)$; curl $\overrightarrow{G} = 2x$.

Conservative?	YES - NO - ???
Potential =	

(b) $\overrightarrow{H} = (e^z, z, y + xe^z)$; curl $\overrightarrow{H} = (0, 0, 0)$.

Conservative?	YES - NO - ???
Potential =	

(c) $\overrightarrow{K} = \left(\frac{1}{x-y}, \frac{1}{y-x}, 2z\right)$; curl $\overrightarrow{K} = (0, 0, 0)$.

Conservative?	YES - NO - ???
Potential =	