Name: ________________________________ **Student Number:** _______________

Math 324B **Advanced Multivariable Calculus I** **Spring 1997**

TEST #2: 4/18/97

- There are 5 problems at 20 points each, for a total of 100 points.
- You may use two $8\frac{1}{2}'' \times 11''$ one-sided pages (or one sheet written on both sides) of your own handwritten notes (no photocopied or printed material).
- *Show your work:* If your work demonstrates that you had the right idea, you might get partial credit even if your answer is wrong. On the other hand, if your answer is not justified by your work, you might lose points even if your answer is correct. If you need more room, continue your work on the back of the page.
- Use the boxes for answers when possible. Measurements must be made with a ruler, and lines and vectors must be drawn with a ruler.
- Simplify algebraic and numerical expressions in answers to the point where they can be evaluated by a student in a precalculus course with a calculator, but you do not need to take them any further. Leave answers such as $\sqrt{7+(6.8)^2}$ as they are, but work out vector expressions such as dot products and cross products. Calculators are allowed, but you won't need one; it is preferable to leave numerical answers in unsimplified form. Laptop computers are not allowed.
- If you have a question, raise your hand and I will come to your seat.

For grader's use only

1:	
2:	
3:	
4:	
5:	
Total:	

1. We consider the vector field $\overrightarrow{G}$ in space given by $\overrightarrow{G}(x, y, z) = (2, x, z)$.

(a) (5 points) The curve C with component functions $(x(t), y(t), z(t))$ is a flow line of $\overrightarrow{G}$. Write down the system of ordinary differential equations that the functions $x(t)$, $y(t)$, and $z(t)$ must satisfy.

ODE: $\boxed{}$

(b) (15 points) Find the equation of the flow line that passes through the point $(2, 1, 1)$ at time $t = 2$.

$x(t) = \boxed{}$

$y(t) = \boxed{}$

$z(t) = \boxed{}$

2. The scalar function $f = f(x, y, z)$ and the vector field $\overrightarrow{G}$ are defined as follows:

$$f(x, y, z) = xe^y - x^2 z; \qquad \overrightarrow{G} = (xy, yx^2, x^2 z^3).$$

Compute the following expressions; if the expression does not make sense, write BAD QUESTION.

(a) (5 points) Compute $\operatorname{div}(\nabla \times \overrightarrow{G})$.

$\operatorname{div}(\nabla \times \overrightarrow{G}) =$

(b) (5 points) Compute $\nabla \cdot (\operatorname{grad} f)$.

$\nabla \cdot (\operatorname{grad} f) =$

(c) (5 points) Compute $\operatorname{curl}(\nabla \cdot \overrightarrow{G})$.

$\operatorname{curl}(\nabla \cdot \overrightarrow{G})$

(d) (5 points) Compute $\operatorname{grad}(\operatorname{div} \overrightarrow{G})$.

$\operatorname{grad}(\operatorname{div} \overrightarrow{G}) =$

3. Let $\overrightarrow{F}$ be the vector field in space defined as follows. At a point P different from the origin O, $\overrightarrow{F}(P)$ is the vector in the same direction as $\overrightarrow{OP}$ with magnitude $\frac{1}{2}$ cm. Let C be the flow line of $\overrightarrow{F}$ that passes through the point $R(3, 0, 4)$ at $t = 0$.

(a) (10 points) Let Q be the point on the flow line C corresponding to the time $t = 1$ second. Find Q.

$Q :$ $\boxed{}$

(b) (10 points) Find a parameter representation for C.

$x(t) =$ $\boxed{}$

$y(t) =$ $\boxed{}$

$z(t) =$ $\boxed{}$

4. Suppose we know the following about a vector field $\overrightarrow{F}(x, y, z)$.

(i) $\overrightarrow{F} = \operatorname{grad} f$ for some function $f = f(x, y, z)$.

(ii) The flow line C passing through $P(0, 0, 1)$ at time $t = 0$ has the parameter representation $(x(t), y(t), z(t)) = (4t, t^2 - t, t^3 + 1)$.

Answer the following questions. If there is not enough information to answer the question, write IMPOSSIBLE.

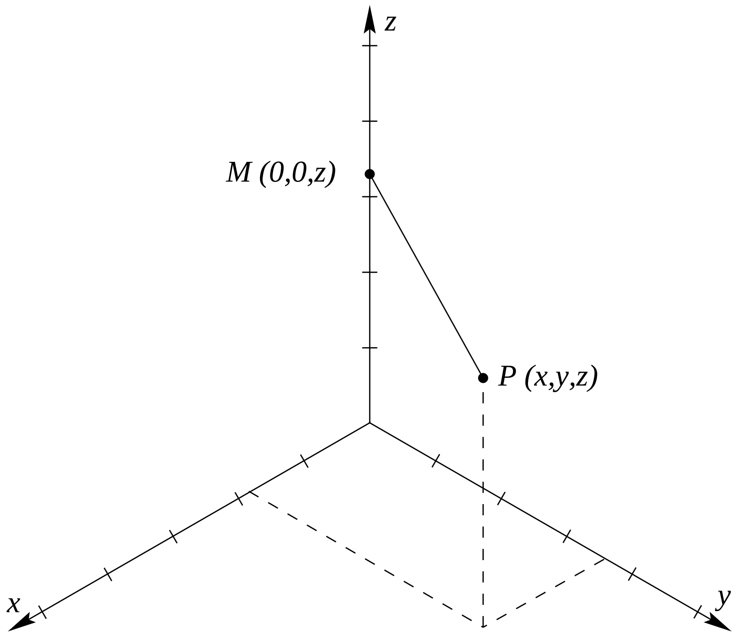
(a) (10 points) Find the field vector $\overrightarrow{F}(0, 0, 1)$ at $P(0, 0, 1)$.

$\overrightarrow{F}(0, 0, 1) = \boxed{}$

(b) (10 points) Let K be the level surface of $f(x, y, z)$ through $P(0, 0, 1)$. Suppose α is the tangent plane to K at $P(0, 0, 1)$. Find an equation for α.

α: $\boxed{}$

5. The vector field $\overrightarrow{F}$ in space is defined as follows: at the point P, the vector $\overrightarrow{F}(P)$ has the same direction as $\overrightarrow{MP}$ and magnitude 3, where M is the point on the z axis at the same height as P. A generic point P and the corresponding point M are shown below.



(a) (8 points) Draw the field vector $\overrightarrow{F}(Q)$ at $Q(2,0,2)$ and label it clearly. Remember that the initial point of this vector should be at Q.

(b) (12 points) Write out the formulas for the component functions $(F_1(x,y,z), F_2(x,y,z), F_3(x,y,z))$ of the vector field.

$F_1(x,y,z) =$ $\boxed{}$

$F_2(x,y,z) =$ $\boxed{}$

$F_3(x,y,z) =$ $\boxed{}$