Math 324A **Advanced Multivariable Calculus I** **Spring 1997**

TEST #4 SOLUTIONS

Outcome: Median for 324A & B: 75 (3.0). Breakdown: 90–100 (4.0): 21%; 75–89 (3.0–4.0): 29%; 60–74 (2.0–3.0): 25%; 45–59 (1.0–2.0): 15%; 0–44 (0.0): 10%.

Answers:

#1 (average score 17/20)

(a) To use the standard parametrization for a plane $\vec{r}(u,v) = \overrightarrow{OA} + u\overrightarrow{AB} + v\overrightarrow{AC}$, we must set $A = R$, $B = Q$, and $C = P$. Therefore the correct parametrization is $(x,y,z) = (0,0,a) + u(0,2a,-a) + v(3a,0,-a)$, or

$$\begin{aligned} x &= 3av \\ y &= 2au \\ z &= a - au - av. \end{aligned}$$

(b) Because the vector field is constant, the flux is $F_n\, A(S)$. The normal vector is $\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC} = (-2a^2, -3a^2, -6a^2)$ (which points in the correct direction) and its length is $a^2\sqrt{4+9+36} = 7a^2$. Therefore the area of the triangle is $\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{7}{2}a^2$, and $F_n = \vec{F} \cdot \vec{n}/|\vec{n}| = (3,2,0)\cdot(-2a^2,-3a^2,-6a^2)/7a^2 = -12/7$. Finally, the flux is $(-12/7)(\frac{7}{2}a^2) = -6a^2$.

#2 (average score 14/20)

(a) The coordinate curve is just given by the parametrization with v set equal to the constant value 2; therefore $(x,y,z) = (2u, 2-u, 4)$. (If you prefer, you may name the parameter t instead of u.) Note that finding the coordinate curve is *not* the same as finding the tangent line.

(b) This is easy if you notice that $\vec{r}_u$ and $\vec{r}_v$ are both vectors tangent to the plane. We compute $\vec{r}_u = (v,-1,0) = (2,-1,0)$ at Q, and $\vec{r}_v = (u,1,2v) = (1,1,4)$ at Q. The point Q has coordinates $(x,y,z) = (2,1,4)$. Therefore the parametrization of the tangent plane is

$$(x,y,z) = (2,1,4) + p(2,-1,0) + q(1,1,4).$$

#3 (average score 12/20)

(a) For a cylinder centered on the z-axis, the normal vector can always be taken as $\vec{n} = (x,y,0)$. Then $F_n = \overrightarrow{F} \cdot \vec{n}/|\vec{n}| = (x^2z^2 + y^2z^2)/\sqrt{x^2+y^2} = z^2\sqrt{x^2+y^2}$. Since $x^2+y^2 = b^2$ on the cylinder, this is just $F_n = z^2 b$.

(b) The maximum value of F_n is attained when $z = a$, and it is $\operatorname{Max} F_n = a^2 b$; the minimum $\operatorname{Min} F_n = 0$ is attained when $z = 0$. Therefore $c = \operatorname{Min} F_n\, A(S) = 0$, and $C = \operatorname{Max} F_n\, A(S) = 2\pi a^3 b^2$.

#4 (average score 17/20)

(a) We compute $\vec{r}_u = (1, 0, -1)$ and $\vec{r}_v = (0, 1, 3)$. Therefore the integrand is

$$\overrightarrow{F}(\vec{r}(u, v)) \cdot (\vec{r}_u \times \vec{r}_v) = (4v, 2u, 3v - 3u) \cdot (1, -3, 1) = 7v - 9u.$$

By constant-u slices, the answer is

$$\int_{u=0}^{u=3} \int_{v=0}^{v=u} (7v - 9u)\, dv\, du.$$

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$$\int_{v=0}^{v=3} \int_{u=v}^{u=3} (7v - 9u)\, du\, dv.$$

#5 (average score 13/20)

(a) Following the hint, we compute the normal component before bothering to set up any parametrizations. The normal component of $\overrightarrow{F}$ is $F_n = (3, 5y^2, 5xyz) \cdot (-1, 0, 0) = -3$. Since this is constant, the flux is $F_n\, A(R) = (-3)(12) = -36$.

(b) Again, you should compute the normal component before trying to write down a parametrization. The normal to a sphere can always be taken to be (x, y, z), so we have $G_n = \overrightarrow{G} \cdot \vec{n}/|\vec{n}| = (5y, -5x, 0) \cdot (x, y, z)/|\vec{n}| = (5yx - 5xy)/|\vec{n}| = 0$. Therefore the flux must be zero.