Math 324B **Advanced Multivariable Calculus I** **Spring 1997**

TEST #1 SOLUTIONS

Outcome: On the whole, the tests were very good. The median score for my two sections combined was 80 out of 100 (equivalent to a grade of approximately 3.3). Here's a more detailed breakdown, showing what percentage of students received each grade: 90–100 (4.0): 34%; 75–90 (3.0–4.0): 33%; 60–75 (2.0–3.0): 12%; 45–59 (1.0–2.0): 13%; 0–44 (0.0): 8%. Average scores on problems (out of 20): Problem 1: 15.4; Problem 2: 17.5; Problem 3: 15.5; Problem 4: 16.3; Problem 5: 12.1.

If your score was less than 45, you are almost certainly not prepared for this course and I recommend dropping it. A score between 45 and 55 is borderline, and you will have to work very hard to succeed. If you want to stay in, I recommend purchasing a copy of the Math 126 notes from Professional Copy and doing problems in it (while keeping up with the work in 324, of course).

Answers: **#1** (a) $P(3,-1,0)$ (b) $Q(0,1,4)$ (c) The equation for l is $(x,y,z) = \overrightarrow{OP} + t\overrightarrow{PQ}$, or $(x,y,z) = (3,-1,0) + t(-3,2,4)$. Other correct answers are possible, but they must give $(3,-1,0)$ when $t = 0$.

#2 (a) By measurement, we have $A(2,0,3)$, $B(3,2,1)$ and $C(1,2.5,2)$. So $\vec{v} = \overrightarrow{AB} = (1,2,-2)$ and $\vec{w} = \overrightarrow{AC} = (-1,2.5,-1)$, and therefore $\vec{v}\cdot\vec{w} = 1\cdot(-1)+2\cdot 2.5+(-2)\cdot(-1) = 6$.
(b) We wish to find c so that $c\vec{v}\cdot\frac{\vec{w}}{|\vec{w}|} = -1$. Since $\vec{v}\cdot\vec{w} = 6$ and $|\vec{w}| = \sqrt{1+(2.5)^2+1}$, we get $c = -\frac{1}{6}\sqrt{2+(2.5)^2}$.

#3 (a) The vectors $\overrightarrow{AB} = (-4,0,3)$ and $\vec{j} = (0,1,0)$ both lie in α, and so $\overrightarrow{AB}\times\vec{j} = (3,0,4)$ is normal to α. Since $|(3,0,4)| = \sqrt{3^2+4^2} = 5$, we get $\vec{n} = \frac{1}{5}(3,0,4) = (\frac{3}{5},0,\frac{4}{5})$.
(b) A normal for α is $(3,0,4)$, so one equation for α is $3x+0y+4z = D$ for some D. We plug in the point $A(4,0,0)$ and find $D = 12$, so our equation for α is $3x+4z = 12$.

#4 (a) Measuring again, we get $P(2,1,0)$, $Q(1,3,3)$, and $R(0,3,0)$. Thus $\overrightarrow{PQ} = (-1,2,3)$ and $\overrightarrow{PR} = (-2,2,0)$. A vector orthogonal to PQR is then $\vec{v} = \overrightarrow{PQ}\times\overrightarrow{PR} = (-6,-6,2)$.
(b) The area of PQR is half the area of the parallelogram spanned by $\overrightarrow{PQ}$ and $\overrightarrow{PR}$; that is, the area of PQR is $\frac{1}{2}|\overrightarrow{PQ}\times\overrightarrow{PR}|$. We've computed this cross product in (a), so the area of PQR is $\frac{1}{2}|(-6,-6,2)| = \frac{1}{2}\sqrt{(-6)^2+(-6)^2+2^2} = \sqrt{19}$.

#5 (a) We first compute $\vec{r}\,'(t) = (2t,2,3t^2)$. We wish to find a time t_0 and a scalar c so that $c\vec{r}\,'(t_0) = (12,-4,-54)$. Restated, this is $(2ct_0,2c,3ct_0^2) = (12,-4,-54)$. Since $2c = -4$, we must have $c = -2$, and so $t_0 = -3$. Checking, we have $\vec{r}\,'(-3) = (-6,2,27)$, which is parallel to $\vec{w} = (12,-4,-54)$.
(b) Let us first compute $P_0 = \vec{r}(t_0) = \vec{r}(-3) = (9,-5,-2)$. The plane α is normal to the tangent vector $\vec{r}\,'(t_0) = (-6,2,27)$ at P_0. Thus one equation for α is $-6x+2y+27z = D$ for some scalar D. We find D by plugging in $P_0(9,-5,-2)$ to find $D = -6(9)+2(-5)+27(-2) = -118$. Our equation for α, then, is $-6x+2y+27z = -118$.